# DETERMINATION OF ROLLING FRICTION LINK CHAIN GEOMETRICAL AND KINEMATIC PARAMETERS 

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#### Abstract

The article describes theoretical research in dismountable conveyor plate chain rolling friction links. Correlations for determination of the bend radius of the chain axle and plate operation surfaces, the axle characteristic point movement trajectory coordinates as well as for evaluation of the strength have been obtained.


Key words: conveyor chain, rolling friction links, determination of parameters.

## Introduction

Improvement of conveyor chain link resistance and safety is achieved in different ways: application of new materials, improvement of the production technology, decrease of roughness of the operation surfaces as well as thermal processing of the link parts and improvement of oil. Still, the reserves for improvement of the strength of plate chains have not been exhausted yet. Irrespective of the fact that the construction of such chains in the result of research during many years has been improved, the principle of link sliding friction has remained unchanged. In all standard conveyor chains (ГОСТ 558-81, ГОСТ 589-74 etc.) in the slinks only the sliding friction principle has been maintained. Transition from sliding friction to rolling friction in links [1] that allows for considerable reduction of the link parts [2] and increase of the chain coefficient of efficiency is considered to be a progressive kind of improvement of conveyor chain further development.

Plate chains with rolling friction links that are formed by two cylindrical shape axles [3] or one axle [2] are known. In these constructions the operation surfaces of the link parts are formed by the contact plate - cylinder.

## Materials and methods

In the present article theoretical research in dismountable plate chain links if rolling friction occurs between the convex surface of the plate 1 and the concave surface of the axle 2 (Fig. 1) is described. The plate 1 operation surface radius is $R_{1}$, but for the axle $2-R_{2}$. In such links if the plates turn rolling friction occurs and the contact stresses decrease.


Fig. 1. Position of link parts in chain rectilinear motion: 1 - plate; 2 - axle; 3 - plate convex operation surface; 4 - concave axle operation surface; 5 - axle center $O_{2}$ movement trajectory


Fig. 2. Position of link parts in link turning
With the axle 2 rolling without sliding along the operation surface of the plate the surface arches $\mathrm{AM}_{1}$ and $\mathrm{AM}_{2}$ will be equal (Fig. 1);

$$
\begin{equation*}
\text { or } R_{1} \Psi_{1}=R_{2} \Psi_{2} \text {, } \tag{1}
\end{equation*}
$$

where $R_{1}$ and $R_{2}$ - plate and axle operation surface radiuses;
$\Psi_{1}$ and $\Psi_{2}$ - angles corresponding to the arches $A M_{1}$ and $A M_{2}$.
When the axle turns up to the position when the points $M_{1}$ and $M_{2}$ (Fig. 2) coincide, the axle turning angle $\gamma$ will be:

$$
\gamma=\Psi_{1}-\Psi_{2} ;
$$

And considering (1)

$$
\begin{equation*}
\gamma=\Psi_{1} \frac{R_{2}-R_{1}}{R_{2}} . \tag{2}
\end{equation*}
$$

In the borderline position when the contact point $M_{l}$ coincides with the point $B$ (Fig. 2) the axle turning angle will be equal to a half of the chain link maximal turning angle $\varphi_{\text {max }}$ :

$$
\begin{equation*}
\gamma_{\text {max }}=\varphi_{\text {max }} / 2=\pi / z, \tag{3}
\end{equation*}
$$

where $z$ - number of sprocket teeth.
Further the coordinate $x$ of the axle and plate contact point $N_{1}$ at the angle $\gamma=\gamma_{\text {max }}$ and the corresponding angle $\Psi_{1 \text { max }}$ are determined (Fig. 3):


Fig. 3. Determination of angle $\Psi_{1 \text { max }}$

$$
\begin{gather*}
x=\frac{0.25 d^{2}+a\left(a+2 R_{1}\right)}{2\left(a+R_{1}\right)} ;  \tag{4}\\
\Psi_{1 \max }=\arccos \frac{a+R_{1}-x}{R_{1}}=\arccos \left[1-\frac{0.25 d^{2}-a^{2}}{2 R_{1}\left(R_{1}+a\right)}\right], \tag{5}
\end{gather*}
$$

where $a$-half of axle "thickness" in the central cut;
$d$ - axle cylindrical part diameter.
Inserting the expression (3) in the expression (2) and evaluating (5) we can obtain the correlation of the main link parameters and the angle $\varphi_{\text {max }}$ :

$$
\begin{equation*}
\frac{\pi}{z}=\frac{R_{2}-R_{1}}{R_{2}} \arccos \left[1-\frac{0.25 d^{2}-a^{2}}{2 R_{1}\left(R_{1}+a\right)}\right] \tag{6}
\end{equation*}
$$

or

$$
\begin{equation*}
R_{2}=R_{1} \frac{\arccos \left[1-\frac{0.25 d^{2}-a^{2}}{2 R_{1}\left(R_{1}+a\right)}\right]}{\arccos \left[1-\frac{0.25 d^{2}-a^{2}}{2 R_{1}\left(R_{1}+a\right)}\right]-\frac{\pi}{z}} . \tag{7}
\end{equation*}
$$

So that in the expression (7) the denominator should be bigger or equal to zero, then

$$
R_{1}<R_{1 \max },
$$

where $R_{1 \text { max }}$ can be expressed by equation:

$$
\begin{equation*}
R_{1 \text { max }}=\frac{-a+\sqrt{a^{2}+2 \frac{0.25 d^{2}-a^{2}}{1-\cos \left(\frac{\pi}{z}\right)}}}{2} . \tag{8}
\end{equation*}
$$

The coordinates of the contact point $M_{1}$ (Fig. 1):

$$
X_{M_{1}}=a+R_{1}-R_{1} \cos \Psi_{1},
$$

where $\quad Y_{M_{1}}=R_{1} \sin \Psi_{1} ;$

$$
\Psi_{1}=\frac{R_{2}}{R_{2}-R_{1}} \gamma=\frac{R_{2}}{R_{2}-R_{1}} \frac{\varphi}{2},
$$

where $\varphi$ - turning angle of one chain link in relation to the other (next) fixed (immobile) link plates.
Then:

$$
\begin{gather*}
X_{M_{1}}=a+R_{1}-R_{1} \cos \left(\frac{R_{2}}{R_{2}-R_{1}} \frac{\varphi}{2}\right) ;  \tag{9}\\
Y_{M_{1}}=R_{l} \sin \left(\frac{R_{2}}{R_{2}-R_{1}} \frac{\varphi}{2}\right) . \tag{10}
\end{gather*}
$$

The peculiarity of rolling friction links is that in the chain link turning the step does not remain constant but changes a bit. In order to analyse the chain step variation depending on the numerical values of the radiuses $R_{1}$ and $R_{2}$ it is necessary to determine the coordinates of the axle center $O_{2}$ movement trajectory.

From Figure 2:

$$
\begin{gathered}
X_{0_{2}^{\prime}}=a+R_{1}-\left(R_{1}+V_{2}\right) \cos \Psi_{1}-V_{1} \sin \left(\Psi_{1}-\Psi_{2}\right) ; \\
Y_{0_{2}^{\prime}}=\left(R_{1}+V_{2}\right) \sin \Psi_{1}-V_{1} \cos \left(\Psi_{1}-\Psi_{2}\right),
\end{gathered}
$$

where (Fig.1):

$$
\begin{aligned}
& \mathrm{V}_{1}=\left(a+R_{2}\right) \operatorname{tg} \Psi_{2} \\
& V_{2}=\frac{a+R_{2}}{\cos \Psi_{2}}-R_{2}
\end{aligned}
$$

The angle $\Psi_{2}$ is the function of the plate turning angle $\varphi$ :

$$
\Psi_{2}=\frac{R_{1}}{R_{2}} \Psi_{1}=\frac{R_{1}}{R_{2}} \frac{R_{2}}{R_{2}-R_{1}} \frac{\varphi}{2}=\frac{R_{1}}{R_{2}-R_{1}} \frac{\varphi}{2} .
$$

Then:

$$
\begin{gathered}
V_{1}=\left(a+R_{2}\right) \operatorname{tg}\left(\frac{R_{1}}{R_{2}-R_{1}} \frac{\varphi}{2}\right) \\
V_{2}=\frac{a+R}{\cos \left(\frac{R_{1}}{R_{2}-R_{1}} \frac{\varphi}{2}\right)}-R_{2}
\end{gathered}
$$

$$
\begin{gather*}
\Psi_{1}-\Psi_{2}=\frac{\varphi}{2} ; \\
X_{0_{2}^{\prime}}=a+R_{1}-\left(R_{1}+V_{2}\right) \cos \left(\frac{R_{2}}{R_{2}-R_{1}} \frac{\varphi}{2}\right)-V_{1} \sin \frac{\varphi}{2} ;  \tag{11}\\
Y_{0_{2}}=\left(R_{1}+V_{2}\right) \sin \left(\frac{R_{2}}{R_{2}-R_{1}} \frac{\varphi}{2}\right)-V_{1} \cos \frac{\varphi}{2} . \tag{12}
\end{gather*}
$$

The axle is calculated in shear. The dangerous cut is $2 a_{\text {min }}$ (Fig. 4). The squares $A_{1}$ and $A_{2}$ are expressed:


Fig. 4. Determination of axle size $\boldsymbol{a}_{\text {min }}$

$$
\begin{gathered}
A=\frac{\pi d^{2}}{4} ; \\
x=\frac{0.25 d^{2}+a_{\min }\left(a_{\min }+2 R_{1}\right)}{2\left(a_{\min }+R_{1}\right)} ; \\
\alpha_{1}=\arccos \frac{2 x}{d} ; \alpha_{2}=\arccos \frac{a_{\min }+R_{1}-x}{R_{1}} ; \\
A_{1}=\frac{R_{1}}{2}\left(\alpha_{1} R_{1}-\left(a_{\min }+R_{1}-x\right) \sin \alpha_{1}\right) ; \\
A_{2}=\frac{d}{4}\left(\frac{\alpha_{2} d}{2}-x \sin \alpha_{2}\right) .
\end{gathered}
$$

If $R_{1}$ is given, the equation including $a_{\text {min }}$ will be:

$$
\begin{equation*}
\frac{\pi d^{2}}{4}-4\left(A_{1}+A_{2}\right)=\frac{F}{\left[\tau_{c}\right]} \tag{13}
\end{equation*}
$$

where $F$ - chain tension force, N ;
$\left[\tau_{c}\right]$ - shear tension permissible for the axle material, $\mathrm{N} \mathrm{mm}^{-2}$.
In order to state the operative part of the plate shape form hole, the coordinates of the characteristic point of the axle movement trajectory should be determined:

$$
\left.\begin{array}{l}
X_{N i}=X_{0_{2}^{\prime}}+X_{N i^{\prime}} \cos \gamma+Y_{N i^{\prime}} \sin \gamma  \tag{14}\\
Y_{N i}=Y_{0_{2}^{\prime}}-X_{N i^{\prime}} \sin \gamma+Y_{N i} \cos \gamma
\end{array}\right\}
$$

where $i=1,2,3,4$ (signs of the characteristic points of the axle $N_{1}, N_{2}, N_{3}, N_{4}$ (Fig. 1);

$$
X_{N_{1}}=\frac{0.25 d^{2}+a\left(a+2 R_{1}\right)}{2\left(a+R_{1}\right)} ; Y_{N_{1}}=\sqrt{0.25 d^{2}-X_{N_{1}}^{2}}
$$



Fig. 5. Determination of movement trajectory of axle characteristic points

$$
\begin{gathered}
X_{N_{2}^{\prime}}=-X_{N_{1}^{\prime}} ; Y_{N_{2}^{\prime}}=Y_{N_{1}^{\prime}} \\
X_{N_{3}^{\prime}}=-X_{N_{1}^{\prime}} ; Y_{N_{3}^{\prime}}=-Y_{N_{1}^{\prime}} \\
X_{N_{4}^{\prime}}=X_{N_{1}^{\prime}} ; Y_{N_{4}^{\prime}}=-Y_{N_{1}^{\prime}}
\end{gathered}
$$

The axle characteristic point movement trajectory coordinates $X_{N i}$ and $Y_{N i}$ are determined at different plate turning angle $\varphi$ values.

## Results and discussion

The rolling friction link part parameters are determined in the following order:

1. Solving the equations ( 8 ) and (13) together the sizes $a_{\min }$ and $R_{1 \text { max }}$ are determined;
2. It is assumed that $R_{1}<R_{\text {max }}$ and again from the equation (13) a is determined;
3. From the equation (7) the radius $R_{2}$ is determined;
4. Knowing $a, R_{1}$ and $R_{2}$ the tension of the socket resistance in the joint axle - plate $\left(\sigma_{H} \leq\left[\sigma_{H}\right]\right)$ is tested;
5. The coordinates of the axle center movement trajectories are determined according to (11) and (12);
6. The trajectory coordinates of the characteristic point of the axle $N_{1}$ are determined according to (14);
7. Make sure whether in the link rolling will be without slipping (research is still necessary).

Assuming different numerical values of the radius $R_{1}\left(R_{1}<R_{1 \max }\right)$ the optimal link parameters are determined.

## Conclusions

1. The contact of the axle and plate operation surfaces has been formed so that the bending center of both surfaces is at one side in the result of what the tension of the socket resistance in the link considerably decreases.
2. The geometrical parameters of the chain axle are determined considering its strength.
3. In the result of the theoretical research correlations are obtained for development of a construction of dismountable plate chain with rolling friction links.

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