# MATHEMATICAL MODEL OF CARROT SLICES DRYING

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**Abstract.** Drying of carrot slices was investigated in a laboratory dryer with a heater with thermostat. Carrots were grated in slices with a thickness of 1-2 mm, width of 4-5 mm and placed in the perforated container with layer thickness 30-35 mm. Based on the experimental data, the time dependency of the moisture content and drying rate were calculated and presented. Using drying rate, the layer of carrot slices drying was simulated and the received results compared with the experimental data. The obtained measurement results are in high correlation with calculations. The presented mathematical model contains a system of partial differential equations including the matter and environmental temperatures ($\Theta(x, t)$ and $T(x, t)$) and mass exchange ($W(x, t)$, $d(x, t)$) and it can be used for thick, porous medium layers, containing small particles, drying. The experimental and theoretical results showed that the carrot slices 3.5 cm thick layer was dry in eight hours using a convective airflow with temperature 37 ºC.

**Keywords:** drying, carrot, mathematical modeling.

## Introduction

Carrot is one of the most common used vegetables for human nutrition due to the high vitamin and fibre content. Carrots have a variety of health effects; they are rich in vitamins, sugar, starch, potassium, calcium, phosphorus, iron and other nutrients and inorganic salts and five kinds of essential amino acids. Since higher temperature causes wilt and have a poor appearance on the carrots, refrigeration and controlled atmosphere storage have been used.

Another way for storage is to dry carrots and then store. The drying process is the use of products with low water activity, thereby inhibiting the production of microbial reproduction and enzyme activity, and can give the flavor of a good product to achieve long-term storage, easy to transport, easy to consumer spending. During drying, heat is supplied and the volatile component, mainly water, is eliminated from the material mixture. During convective drying, two entirely different processes take place:

- elimination of water from the surface by warm air;
- diffusion of water from within towards the surface due to the concentration difference.

Many studies were done to process carrots by air drying [1], sun drying [2] and solar drying [3]. Several researches have been done to influence some process parameters (temperature, sample thickness, air flow rate, etc). The effect of carrot slices on the drying kinetics was studied in [4]. The modeling of carrot cubes was made in [5]. The authors studied influence of air-flow rates and effect of temperature on the drying curve for carrot cubes. Liu Zhenghuai [6] studied the drying process of thick carrot slices heat transfer simulation, integrated heat diffusion process, sliced carrots proposed internal heat transfer model and internal mass transfer model, the use of the third heat transfer boundary conditions were simulated and experimental comparison done.

The aim of this research was to investigate thin carrot slices drying using small heated air and determining the drying coefficient. This paper presents a mathematical model for carrot slices layer drying.

## Materials and methods

Carrots were grated in slices with a thickness of 1-2 mm, width of 4-5 mm. The carrot slices were placed in the perforated container with the layer thickness of 30-35 mm (Fig. 1).

The equipment was manufactured, which allows studying the drying process of carrot slices (Fig. 2). The thermostat allows to maintain constant temperature of the sample layer on one side. The inlet air temperature was 37 ºC. The heated air flow was moving by convection. The container was weighted with electronic instruments to determine the quantity of water runoff during the experiment.

Fig.1. **Sample of carrot slices**

Drying of any substance is based on heat-mass transfer processes. In our situation it is based on heat-mass transfer between the carrot slices and inter-slices space. Since the slice thickness is very small, the internal diffusion in the drying process can be ignored.

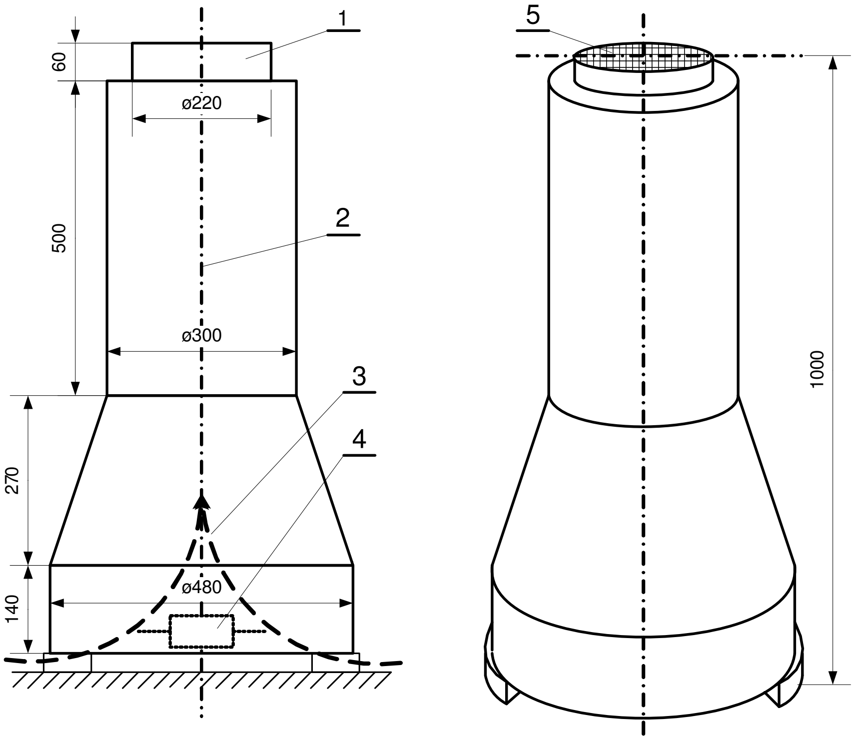


Fig. 2. **Scheme of experimental device:** 1 – outlet air temperature; 2 – inlet air temperature; 3 – inlet air; 4 – heater with thermostat; 5 – sieve with products

We propose a mathematical model which contains temperature and moisture functions of the matter (carrot slices) and inter-matter space (air). To describe the kinetics of the drying process we assume the following:

- water evaporation in slices of carrots proceeds according to Dalton law;
- water is liquid in carrots;
- heat transfer between the matter and drying agent (air) goes on by convection;
- the air flow takes place due to convection and its velocity is constant in the layer of matter;
- inner temperature gradient for a single matter slice is very small and has not been considered.

The heat-mass transfer model is based on laws of physics, i.e., the mass transfer law between the matter and drying agent, the law of substance conservation, the law of heat transfer between the matter and air and the law of energy conservation. We obtained the following system of partial differential equations including the matter and environmental temperatures ($\Theta(x, t)$ and $T(x, t)$) and mass exchange ($W(x, t)$, $d(x, t)$) [7]:

$$\frac{\partial W}{\partial t} = K\left(W_p - W\right),\ \ t > 0, x > 0, \tag{1}$$

$$\frac{\partial d}{\partial t} + a_1 \frac{\partial d}{\partial x} = \frac{k}{a_2}\left(W - W_p\right),\ t > 0, x > 0, \tag{2}$$

$$\frac{\partial \Theta}{\partial t} = c_1\left(T - \Theta\right) + c_2\left(W_p - W\right),\ t > 0, x > 0, \tag{3}$$

$$\frac{\partial T}{\partial t} + a_1 \frac{\partial T}{\partial x} = c_0\left(\Theta - T\right)\ t > 0, x > 0, \tag{4}$$

where $x$, $t$ – variable of space and time.

There $a_1 = 3600v,\ a_2 = \dfrac{\gamma_a \varepsilon}{10\gamma_m},\ c_0 = \dfrac{\alpha_q}{m\gamma_a c_a},\ c_1 = \dfrac{\alpha_q}{(m-1)\gamma_m c_m},$

$$c_2 = \frac{Kr}{100c_g},\ K = \exp(20.95 - \frac{6942}{T + 273}),\ \alpha_q = 12.6\frac{\lambda}{L^2}.$$

The notations are:

$v$ – air velocity, $m \cdot s^{-1}$;
$\gamma_a$, $\gamma_m$ – capacity of weight (air and matter respectively), $kg \cdot m^{-3}$;
$c_a$, $c_m$ – heat of drying air and moist matter, $kJ \cdot kg^{-1}$;
$r$ – latent heat for water evaporation, $kJ \cdot kg^{-1}$;
$\varepsilon = m/(1-m)$, $m$ – porosity of matter;
$W_p$ – equilibrium moisture content, dry basis, %;
$K$ – drying coefficient, $h^{-1}$;
$\alpha_q$ – interphase heat exchange coefficient, $kJ \cdot m^{-3} \cdot h^{-1} \cdot {}^{\circ}C^{-1}$;
$\lambda$ – rate of matter heat transfer, $kJ \cdot m^{-1} \cdot h^{-1} \cdot {}^{\circ}C^{-1}$;
$2L$ – carrot slice thickness, m.

The equilibrium moisture content $W_p$ was obtained using S. Henderson's modified equation in Forte's interpretation :

$$W_p = \left(-\frac{1}{5869}\ln\left(1 - \frac{\varphi}{100}\right)(T + 273)^{0.775}\right)^{\frac{(T+273)^{1.363}}{5203}},$$

where $\phi$ – heated air relative humidity, %.

The initial and boundary conditions for the system (1) - (4) can be given in the following way:

$$T\big|_{t=0} = \Theta\big|_{t=0} = \Psi_1(x)\ ,\ W\big|_{t=0} = \Psi_2(x)\ ,\ d\big|_{t=0} = \Psi_3(x)$$

$$T\big|_{x=0} = \vartheta_1(t)\ ,\ d\big|_{x=0} = \vartheta_2(t)\ . \tag{5}$$

The initial conditions for the system (1)-(4) are given as follows:

$$\Psi_1(x) = \Theta_s\ ,\ \Psi_2(x) = W_s\ ,\ \Psi_3(x) = d_s\ ,$$

where $\Theta_s$ (°C) is the matter and intermatter air temperature in the layer; $W_s$, $d_s$ (%) are the matter moisture and intermatter air humidity in the layer. For carrot chips drying we chose constant boundary conditions:

$$\vartheta_1(t) = T_r\ ,\ \vartheta_2(t) = d_r\ ,$$

where $T_r$ (ºC) and $d_r$ (%) heated air temperature and humidity.

The system (1)-(4) with boundary and initial conditions (5) can be solved numerically by difference scheme using weights [7].

**Results and discussion**

As shown by the experiments the drying coefficient $K$ is not constant but depends on the temperature. As in our case $T_r$ is constant, then the experimental data are used to determine $K$ dependence on the drying time.

As we do not know the real carrot slices moisture $W_s$ and equilibrium moisture $Wp$, we take the experimental data rationing $u(t)$:

$$u(t) = \frac{W(t) - W_p}{W_s - W_p}.$$

It should be noted that in the case of *W (t)* we understand the whole layer of the mean integral moisture. Using equation (1) and condition (5) we expressed the drying coefficient $K$ ($t$):

$$K(t) = -\frac{\ln(u)}{t},$$

where $t$-drying time (min).

Using the processed data the carrot slices drying coefficient was obtained, depending on the drying time

$$K(t) = 300.434 \cdot 10^{-5} + 1.12 \cdot 10^{-5} \cdot t, \tag{6}$$

with the coefficient of determination $\eta^2$=0.99.

Using expression (6), obtained from the experimental data, we solved equation (1). The experimental and numerical results are shown in Fig. 3.

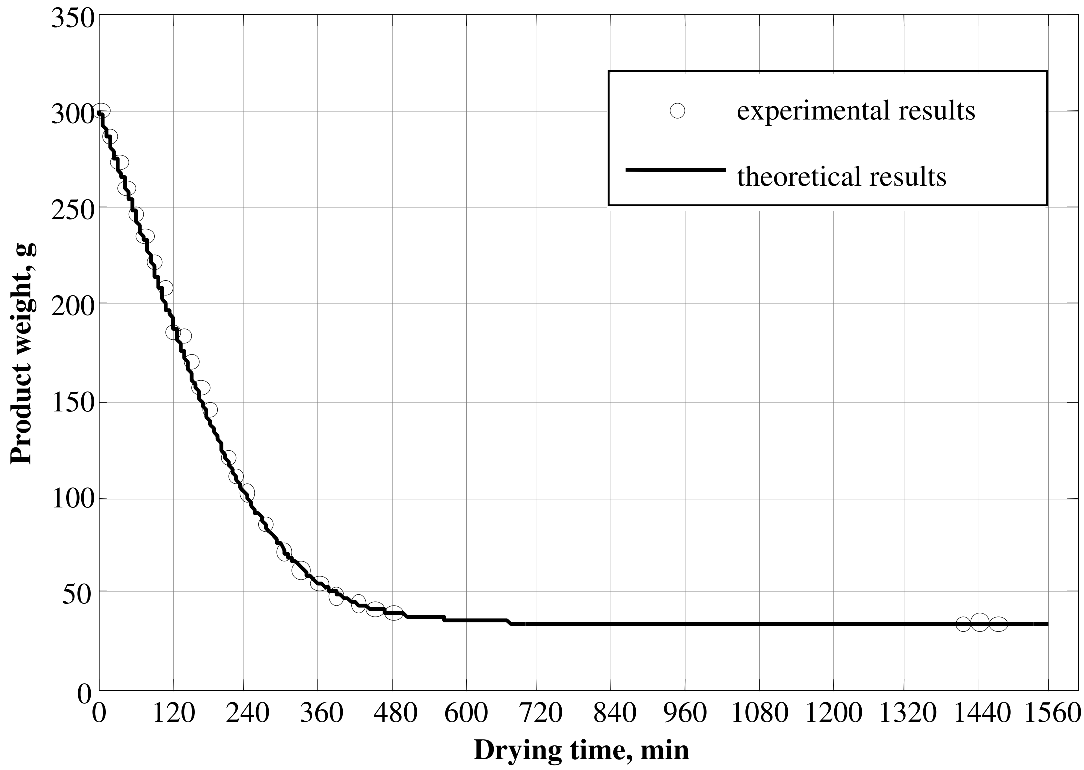


Fig. 3. **Carrot slices weight changes during drying process**

The resulting drying coefficient expression (6) can be used for simulating thick carrot slices layer drying using the system (1)-(4) with initial and boundary conditions (5).

## Conclusions

1. The experimental and theoretical results showed that the carrot slices 3.5 cm thick layer was dry in eight hours using convective airflow with temperature 37 ºC.
2. The offered mathematical model can be used for modeling the drying process of thin carrot slices in a thick layer with the drying agent velocity *v*, where there are differences between the air and matter temperatures. For a thin layer we can use the first equation with changing drying coefficient.

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