SYNTHESIS OF A ROBOT FISH PROTOTYPE

Igors Tipans, Semjons Cifanskis, Janis Viba,
Vladimirs Jakushevich, Bruno Grasmanis, Guntis Kulikovskis
Riga Technical University, Latvia
igors.tipans@rtu.lv, semjons.cifanskis@rtu.lv, janis.viba@rtu.lv,
vladimirs.jakushevich@rtu.lv, bruno.grasmanis@rtu.lv, guntis.kulikovskis@rtu.lv

Abstract. Theoretical analysis of two types of motion is presented. The first type is a motion inside stationary flow. The main task is to find an optimal control law for variation of additional surface area of the working head interacting with water medium. The optimization criterion is the time required to move the object from the initial position to the given end position. The second type of motion was a fin type horizontal pendulum motion and corresponding model optimization. The task was to find an optimal control law for variation of additional area of vibrating horizontal tail which ensures maximal positive impulse of forces acting on a pivot. Both problems were solved by using Pontryagin's minimum principle. It is shown that optimal control actions correspond to the case of "bang – bang" control. Examples of synthesis of real mechatronic systems are given. As a result, a real prototype of robot fish was made. Such prototype was experimentally investigated in linear water tanks to solve different problems, e.g., to find the maximal motion velocity depending on the power pack capacity and to find the minimal propulsion force, depending on the system parameters. The robot fish prototype was investigated in a lake with autonomous power pack and distance control system, moving in real underwater conditions. The results of the theoretical and experimental investigations may be used for development of new underwater robotic systems.

Keywords: underwater robotics, optimization, synthesis.

Introduction

Motion of a vibrator with two degrees of freedom and constant air flow V_0 excitation is investigated (Fig. 1, 2). The system consists of masse m with spring c and damper b.
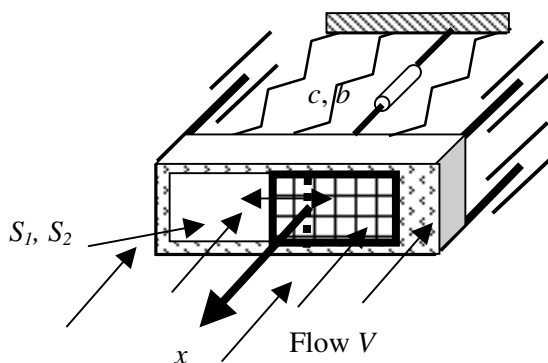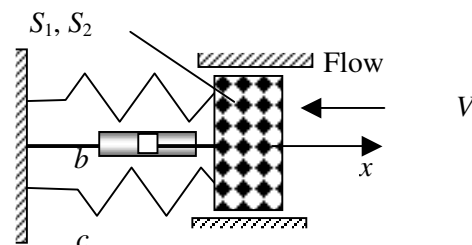

Fig. 1. **Working head construction**   Fig. 2. **Side-view of mathematical model**

The main idea is to find out an optimal control law for variation of additional area $S(t)$ of vibrating mass m within limits (1):

$$S_1 \leq S(t) \leq S_2, \tag{1}$$

where   (here and next International System of Units is used)
    S_1 – lower level of additional area of mass m;
    S_2 – upper level of additional area of mass m;
    t – time.

The criterion of optimization is time T required to move the object from the initial position to the end position.

Then the differential equation is (2):

$$m\,\ddot{x} = -c\,x - b\,\dot{x} - u(t)\cdot(V_0 + \dot{x})^2, \tag{2}$$

where $u(t)=S(t)\cdot k$;

c – stiffness of spring;
b – damping coefficient;
V_0 – constant velocity of water;
$S(t)$ – area variation;
$u(t)$ – control action (2);
k – constant.

It is required to determine the control action $u = u(t)$ for displacement of a system (2) from the initial position $x(t_0)$ to the end position $x(t_1)$ in minimal time (criterion K) $K=T$, if the area $S(t)$ has limit (1).

Solution of optimal control problem for system with one degree of freedom

For system excitation any time the high-speed problem must be solved [1-3]:

$$K = \int_{t_0}^{t_1} 1 \cdot dt \tag{3}$$

To assume $t_0 = 0;\quad t_1 = T$, from (3) we have $K = T$.

The problems have been solved using the maximum principle of Pontryagin. It is find out that optimal control action corresponds to the case of bound values of area limits S_1, S_2 [4-9].

Real control action synthesis

For realizing optimal control actions (in general case) the system of one degree of freedom needs a feedback system with two adapters: one for displacement measurement and another - for velocity measurement. There is a simple case of control existing with only one adapter when motion changes directions [2-4]. It means that the control action is similar to negative dry friction and switch points are along zero velocity line. In that case equation of motion for large velocity $|V0| \geq |\dot{x}|$ is (4):

$$m \cdot \ddot{x} = -c \cdot x - b \cdot \dot{x} + U(\dot{x}), \tag{4}$$

where $U(\dot{x}) = -\left[k \cdot (V_0 + \dot{x})^2 \cdot S_1 \cdot \dfrac{1 + sign(\dot{x})}{2} \right] - \left[k \cdot (V_0 + \dot{x})^2 \cdot S_2 \cdot \dfrac{1 - sign(\dot{x})}{2} \right]$

m – mass;
c, b, F, k, V_0 – constants.

Examples of modelling are shown in Fig. 3-4.

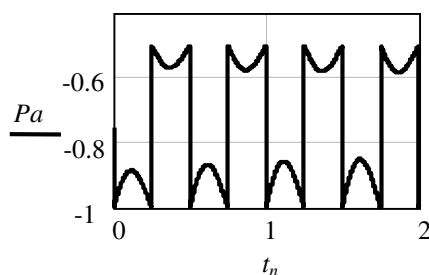$$\begin{pmatrix} x_0 \\ v_0 \end{pmatrix} := \begin{pmatrix} 2.3 \\ 0 \end{pmatrix}$$

Fig. 3. **Full control action $Pa_n=U(\dot{x})$ in time t_n domain (SI system)**
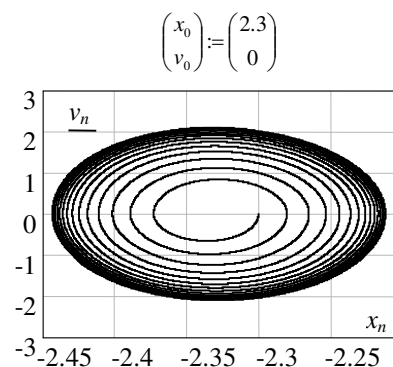
Fig. 4. **Motion in phase plane ($x=x_n$; $\dot{x}=v_n$) with initial conditions inside of limit cycle**

Optimization and synthesis of robot fish tail rotation motion

The second object of the theoretical study is a fin type propulsive device of robotic fish moving inside water. The aim of the study is to find out an optimal control law for variation of additional area

of the vibrating tail like horizontal pendulum, which ensures maximal positive impulse of motion forces components acting on the tail parallel x axe (Fig. 5, 6).
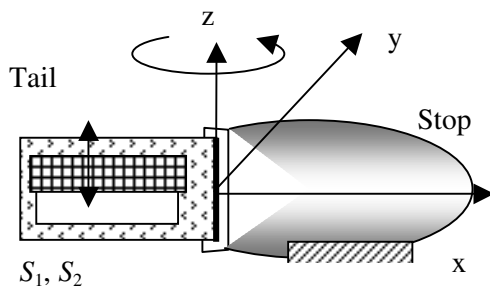

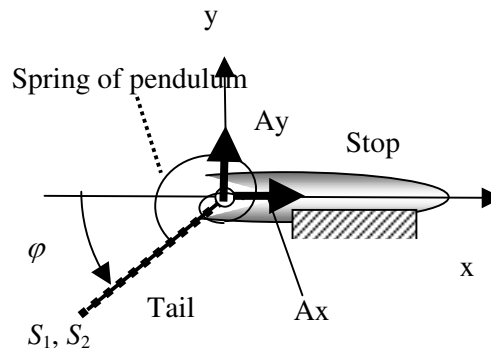Fig. 5. **Side-view of robot fish tail like horizontal pendulum**

Fig. 6. **Top-view of robot fish tail like horizontal pendulum. Simplify model with fixed pivot A**

The problem has been solved using the maximum principle of Pontryagin. It is shown that the optimal control action corresponds to the case of bound values of area limits.

The mathematical model of the system consists of rigid straight flat tail moving around the pivot (Fig. 6). The area of the tail may be changed by control actions. For motion excitation moment $M(t,\varphi,\omega)$ around the pivot must be added, where $\omega=d\varphi/dt$ – angular velocity of rigid tail. For more resources of the system an additional rotational spring with stiffness c may be added (Fig. 6).

The main idea of this report is to find out an optimal control law for variation of additional area $B(t)$ of the vibrating tail within limits (5):

$$B_1 \le B(t) \le B_2,\qquad(5)$$

where B_1 – lower level of additional area of tail;
B_2 – upper level of additional area of tail;
t – time.

The criterion of optimization is full reaction Ax^* impulse in the pivot, acted to hull (indirect reaction to tail, Fig. 6.):

$$K = -\int_0^T Ax^* \cdot dt,\qquad(6)$$

required to move the object from one stop initial position ($\varphi_{max};\ \dot{\varphi}=0$) to the second stop end position ($\varphi_{min},\dot{\varphi}=0$) in time T.

The differential equation of motion of the system with one degree of freedom (by use exchange of moment of momentum around a fixed point A) is:

$$J_A\ddot{\varphi} = M(t,\varphi,\dot{\varphi}) - c\cdot\varphi - k_t\cdot B\cdot sign(\dot{\varphi}\cdot\varphi)\cdot\int_0^L (\dot{\varphi}\cdot\xi)^2\cdot\xi\cdot d\xi,\qquad(7)$$

where J_A – moment inertia tail mass against pivot point;
$\ddot{\varphi}$ – angular acceleration of rigid body straight tail;
$M(t,\varphi,\dot{\varphi})$ – excitation moment of drive;
c – angular stiffness;
k_t – constant coefficient;
B – area;

$(\int_0^L (\dot{\varphi} \cdot \xi)^2 \cdot \xi \cdot d\xi)$ – component of moment of resistance force like integral along tail direction;

$\dot{\varphi}$ – angular velocity;

L – length of the tail.

From theorem of tail mass m centre motion follows (8) (Fig. 6.):

$$m \cdot \left(\dot{\varphi}^2 \cdot \frac{L}{2} \cdot \cos\varphi + \ddot{\varphi} \cdot \frac{L}{2} \cdot \sin\varphi \right) = Ax - k_t \cdot B \cdot \sin(\varphi) \cdot sign(\varphi \cdot \dot{\varphi}) \cdot \left(\int_0^L (\dot{\varphi} \cdot \xi)^2 \cdot d\xi \right) \tag{8}$$

After integration from equations (6) and (7) we have (9, 10):

$$\ddot{\varphi} = \frac{1}{J_A} \cdot \left[M(t, \varphi, \dot{\varphi}) - c \cdot \varphi - k_t \cdot B \cdot sign(\dot{\varphi}) \cdot \dot{\varphi}^2 \cdot \frac{L^4}{4} \right] \tag{9}$$

$$Ax = m \cdot \{ \dot{\varphi}^2 \cdot \frac{L}{2} \cdot \cos(\varphi) + \frac{1}{J_A} \cdot \left[M(t, \varphi, \dot{\varphi}) - c \cdot \varphi - k_t \cdot B \cdot sign(\dot{\varphi}) \cdot \dot{\varphi}^2 \cdot \frac{L^4}{4} \right] \cdot \frac{L}{2} \cdot \sin(\varphi) \} +$$

$$+ k_t \cdot B \cdot \sin(\varphi) \cdot sign(\varphi \cdot \dot{\varphi}) \cdot \dot{\varphi}^2 \cdot \frac{L^3}{3}. \tag{10}$$

Then the criterion of optimization (fool impulse) is:

$$K = -\int_0^T \left(\begin{array}{c} m \cdot \{ \dot{\varphi}^2 \cdot \frac{L}{2} \cdot \cos(\varphi) + \frac{1}{J_A} \cdot \left[M(t, \varphi, \dot{\varphi}) - c \cdot \varphi - k_t \cdot B \cdot sign(\dot{\varphi}) \cdot \dot{\varphi}^2 \cdot \frac{L^4}{4} \right] \cdot \frac{L}{2} \cdot \sin(\varphi) \} + \\ + k_t \cdot B \cdot \sin(\varphi) \cdot sign(\varphi \cdot \dot{\varphi}) \cdot \dot{\varphi}^2 \cdot \frac{L^3}{3} \end{array} \right) \cdot dt. \tag{11}$$

The last equations (11) can be used in the task of optimizations. The solution of optimal control problem for the system with one degree of freedom (9) by using the maximum principle of Portryaing may be found out from equations (6, 7) [6-10]. Like before, the control action is on the bounds boundaries (5).

Synthesis of system with adaptive excitation and adaptive area control

In a case of adaptive excitation moment M may be used like a function of angular velocity in a form [3; 9] $M(t) = M0 \cdot sign(\omega)$. The results of modelling are shown in Fig. 7-10.
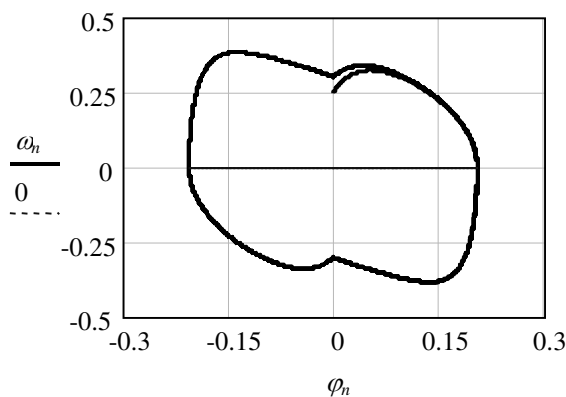

Fig. 7. **Tail angular motion in phase plane – angular velocity as function of angle**
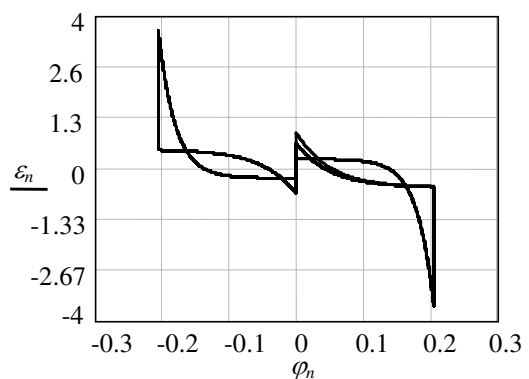

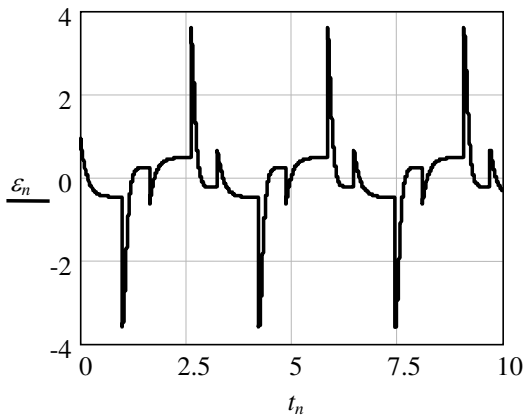Fig. 8. **Angular acceleration of tail as function of angle**

Fig. 9. **Angular acceleration $\varepsilon = \ddot{\varphi}$ of tail in time domain**
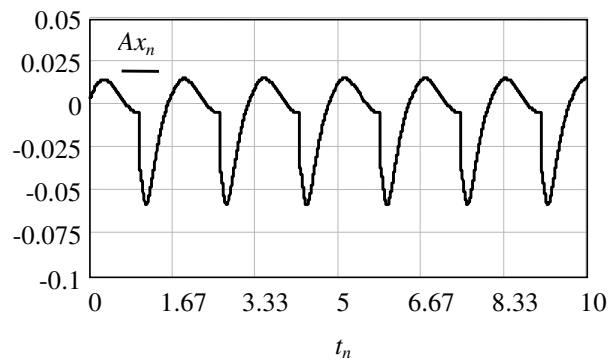

Fig. 10. **Impulse $Ax(t)$ in time domain**

A prototype of robot fish for experimental investigations was made (Fig. 11, 12). This prototype was investigated in a linear water tank with different aims, for example: – find maximal motion velocity depending on the power pack capacity; – find minimal propulsion force, depending on the system parameters.

Additionally this prototype was investigated in a large storage lake with autonomy power pack and distance control system, moving in real under water conditions with waves. The results of the theoretical and experimental investigation may be used for inventions of new robotic systems.
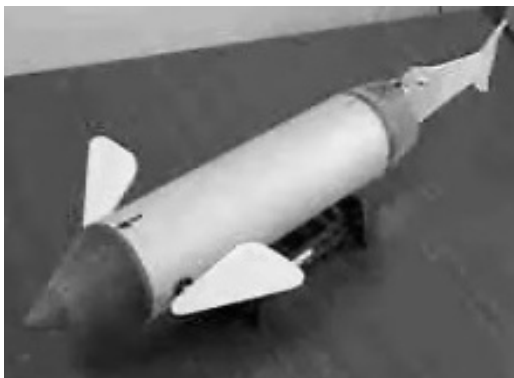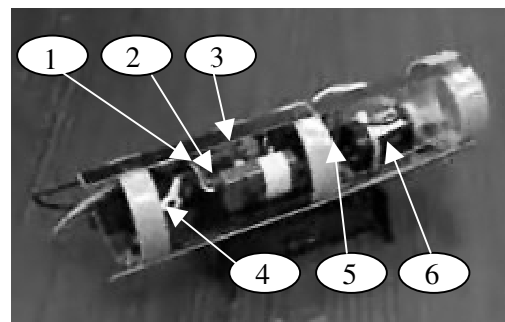

Fig. 11. **Top-view of prototype**

Fig. 12. **Inside prototype there are:**
1 – microcontroller; three actuators
(4 – for level, 5 – direction and 6 – velocity control);
2 – power supply; 3 – radio signal detector

Conclusions

Motion of robotic fish tail vibration by simplified interaction with water flow give very easy equations of motion analysis. That allows solving the mathematical problem of area control optimization and gives information for new systems synthesis. Control (exchange) of the object area under water allows finding very efficient mechatronic systems. For realization of such systems adapters and controllers must be used. For this reason very simple control action has solutions with use of sign functions. It is shown that real tail vibration motion is very stable and gives satisfactory real criterion in the hull contact point.

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