MODEL ANALYSIS OF TEMPERATURE SENSORS

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Abstract. A complete substitute model of the measured object, including sensors arranged in the same circuit is described in the article. The mutual influences of individual parts of an equivalent circuit can be characterized by several real situations by connecting the sensor to the measured system and the mutual influence of both objects (the sensor and the measured object), interference signal by external application of sources and the mistake of transformation input variables to the output variable, the action of dynamic error. The models have a character ladder of networks in combination with RC elements. It is necessary to create differential equations of the object and the measuring chain for implementation mathematical analysis. In this case (in simpler systems), we can only use algebraic computation. The time course of warming can be analyzed using different models together with application of correction terms of the analyzed signal. After analyzing the dependence of the measured object and the sensor, it is another part of the work of connection the object and sensor in a common system. In the first part there will be an analysis without a corrector. The common system is desirable only in the cases, where we are able to assemble such a corrector, the dynamic characteristics of which we know. This is due to the possible distortion waveforms. In the resulting scheme it is also necessary to complement the character of heat transfer, conduction of heat supply conductors and possible waveform distortion of the overall by noise. It usually has a stochastic character. The outcomes of the work can be applied for the possibility of creating a model of the measurement object and measuring system.

Keywords: model, Matlab, analysis, temperature, correction model.

Introduction

System modelling is one of the options for obtaining qualitative information about the system properties. Its advantages are emphasized mainly in cases where parameters of the given process can not be determined by the classical approach, i.e. when it is not possible to accurately measure the static and dynamic characteristics of the physical object. There are several reasons why this is not possible. An experiment on the physical object may cause its damage, accident, destruction, production decrease, or it is difficult to measure the characteristics due to inappropriate measuring techniques.

There are two methods for mathematical modelling. The first of these is the analytical method of model formation. It is based on material and energetic balance of the physical device and on the knowledge of physical and chemical processes taking place. The model describes inner status values and their link. It is a behavioural model of the system, where the output is a state-model of the object (internal model). The second of these is the empirical method. It is based on measuring input and output values on the physical object and their evaluation, where the output is an experimental model of the physical system. This model is not a structural model of the system, it circumvents the inner status values. It is only a behavioural model, it shows only the links between input and output [1].

When applied in practise, it is important to know temperature dependencies of the objects used not only in electrical engineering, but also the measuring elements, which can significantly affect the presentation of the measured quantity. Each object is presented with a parameter of load capacity, which usually determines the maximum heat loss by the passage of electric current through the object. Poorly designed power switch circuits may result in a change in specific parameters of the component [2; 3]. During measuring the waveform the rapid temperature change leads to phase shift of real temperature from the ideal temperature. During measuring the waveform the slow temperature changes, in range of seconds, lead not to big difference of temperature.

Materials and methods

The issues of the model development are based on the principle of analogy among mechanical, hydraulic, pneumatic and electrical circuit. Such a "substitute" electrical circuit, so electrical analogy, is presented with a specific transitional effect.

We take an approach of certain simplification when analysing the subsequent problems, which consists in working with models with lumped parameters. We assume that the even temperature distribution and temperature change speed are proportional to the applied heat (1) [4].

$$\rho \cdot c_p \cdot V \cdot \frac{dT}{dt} = \Phi_p + \Phi_v + \Phi_s , \tag{1}$$

where ρ – density of the analyzed object, kg·m^{-3};
V – volume of the analyzed object, m^3;
c_p – specific heat capacity, J·kg^{-1}·K^{-1};
Φ – heat flux, Φ_p convection, Φ_v conduction, Φ_s radiation, W.

Analogous induction is not taking place in thermal systems where the second law of thermodynamics applies. It means that there are responses with an aperiodic waveform during spontaneous heat transfer.

Assume that heat is generated at one point by a miniature resistor, which is heated by electrical current. Another assumption is sufficient heating time so that there is no further temperature change of the object.

Temperature field h is radial and spherically symmetric. The surface area through we use integral is $4\pi R^2$. As per (2-3) h is a vector length $\left|\vec{h}\right|$. Stated integral is supposed to be equal to the speed of heat creation, this speed is determined as P [4].

$$\oint_S h \cdot n \cdot dS = h \cdot 4\pi R^2 , \tag{2}$$

$$h = \frac{P}{4\pi R^2} , \tag{3}$$

where h – amount of heat during temperature gradient in time unit, J·m^{-2}·s^{-1};
$\vec{n}$ – unit normal vector to the surface, m^2;
S – affected surface, m^2;
R – radius of the radial flow of a point source of, m.

Per the stated formula h is directly proportional to P and inversely proportional to the square of the distance from the source. The stated dependence is suitable for applications of heat flux near the point source. The differential equation of Fourier heat transfer under the condition of its maintaining (4-11) [4].

$$h = -\lambda \nabla T , \tag{4}$$

$$-\frac{dq}{dt} = \nabla \cdot h = -\nabla(\lambda \nabla T) , \tag{5}$$

$$\frac{dq}{dt} = \lambda \nabla \cdot \nabla T = \lambda \nabla^2 T , \tag{6}$$

where T – thermodynamic temperature, K;
$\frac{dq}{dt}$ – amount of heat per unit volume and unit time, W·m^{-3};
λ – thermal conductivity, W·m^{-1}·K^{-1};
∇^2 – Laplace operator, K·m^{-2};
∇ – gradient operator, defines a derivative of x-, y- and z-components of the vector.

$$\mathrm{grad}\, T = \nabla T = \left(\frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}, \frac{\partial T}{\partial z} \right). \tag{7}$$

Provided that λ is constant, then q is the amount of heat in unit volume and ∇^2 is the Laplace operator.

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}, \quad (8)$$

$$\Delta q = \rho \cdot c_v \Delta T, \quad (9)$$

$$\frac{dq}{dt} = \rho \cdot c_v \frac{dT}{dt}, \quad (10)$$

where c_v – specific heat capacity ($J \cdot kg^{-1} \cdot K^{-1}$).

$$\frac{dT}{dt} = \frac{\lambda}{\rho \cdot c_v} \nabla^2 T. \quad (11)$$

Rate of change *T* is directly proportional to the Laplace operator and second derivative of dependence with respect to position. Differentiate equation of heat diffusion *D* states the thermal diffusion (12).

$$\frac{dT}{dt} = D \nabla^2 T, \quad (12)$$

where D – thermal-diffusion coefficient $\frac{\lambda}{c_v}$, $m^2 \cdot s^{-1}$.

In Fig. 1 a complete substitute model of the measured object is shown as well as the sensor arranged in one circuit. The switch in position 1 represents a state when there is no sensor attached to the object. The original surface temperature is defined by the voltage *E*(*t*). By attaching the sensor to the measured object the switch changes over to position 2. At first, the material of the sensor starts to warm through *Cs* via the transitional resistor *Rp*. It is essential that the product of *Rp. Cs* is as low as possible in order to maintain low time constants. Then, after warming through the sensor's material the heat dispersal into the surroundings starts to take place *Rr*. An ideal condition would be if *E*(*t*) remained at its original value and the value of *U*(*t*) is as close as possible. This would mean that by attaching the sensor to the measured object the thermal field would not be affected. The measurement device shows directly the value *E*(*t*) without a change [5].

Mutual influences of individual parts of the substitute circuit are possible to characterise by several real situations:

- by attaching the sensor to the measured system, and by mutual influence of both objects (sensor and measured object);
- by disrupting the carrier signal via external source;
- transformation errors from an entry quantity to an exit quantity, and an effect of a dynamic error.

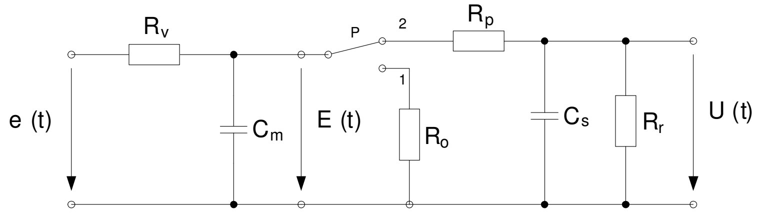

Fig. 1. **Model of mutual influence between the thermocouple and the measured object [6]**

These conditions can be partially met when *Rp* is very low or, on the contrary, the heat dispersal to the surroundings *Rr* very high. The applicable formula is (13-14):

$$R_P << R_N, \quad (13)$$

$$U(t) = E(t) \cdot \frac{R_r}{R_r + R_p}. \quad (14)$$

Condition $U(t) = E(t)$ is met if $Rp = 0$ or $Rr \to \infty$. [6]

In practical cases the ratio of *U/E* is always less than 1, this is based on the physical nature of the sensor. The more is the ratio closer to 1, the more accurate is the measuring circuit, i.e. the measured temperature in a steady state. This condition is not possible to achieve by the effect of thermal capacity and inertia of the system. In the model situations we are able to draw closer to this state, but this is not desirable. Fig. 2 shows the substitute scheme of the object with inertia of a first order. The stated case is an example of the most basic model, which does not occur in a real system. It takes into consideration only the basic character of the system without the real impacts. RC element is linked to the structure of the measuring chain in a connected state of the measuring part. Fig. 3 is a model of the temperature sensor.

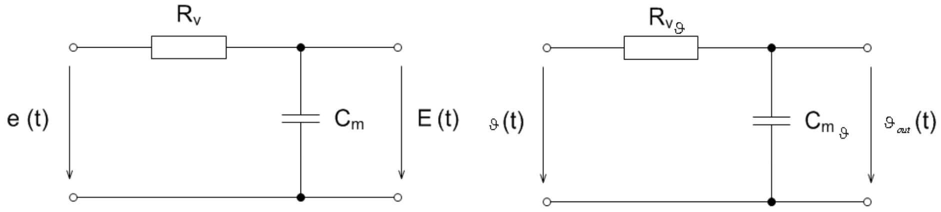

Fig. 2. **Measured object [6]**

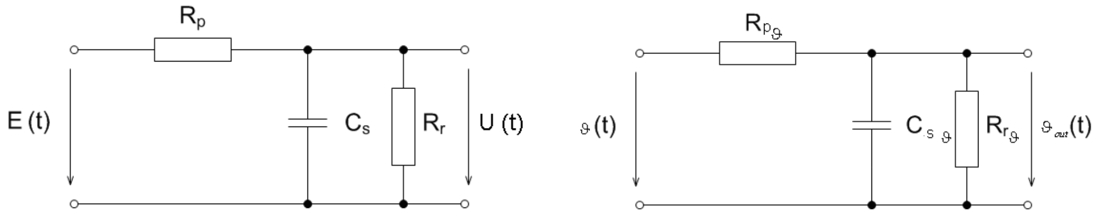

Fig. 3. **Model of sensor [6]**

In the figures the parameters of characteristics such as thermal resistance *R* ($K \cdot W^{-1}$) and thermal capacity *C* ($J \cdot K^{-1}$) are shown, which determine basic links of inertia. *E*(*t*) the original surface temperature of the component without attaching the sensor, in case the circuit is supplemented with the elimination resistor *Rr*. *E*(*t*), in the second model, there is the input signal of the temperature sensor circuit. *U*(*t*) is the measured output signal. *Cs* is proportional to the sensor mass. *Rp* is the transitional resistor and *Rr* heat dispersal to the surroundings.

Results and discussion

Types of models have a character of a ranking net in RC elements combination. The basic and simple structure is pictured in Fig. 4. According to the pictured model of the object a link between the sensor attachment to the measured object and the switch changing from position 1 to position 2 are apparent.

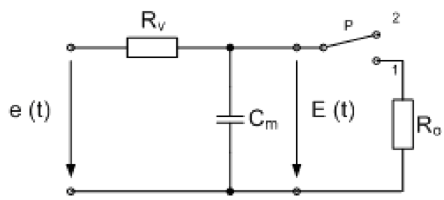

Fig. 4. **Model of the object supplemented with R_0**

If the switch is in position 1, there is heat radiation to the surroundings and the RC structure is supplemented with another resistor. This structure might be variously supplemented according to time-related RC analogues. We started to converse solutions to systems of the first order differential equations, and subsequently to the second order differential equations, where the characteristic arrangement is realized by a *T* – element, π – element and by a crossbar element, the option depends on the type of the system [7].

The measuring circuit is attached in position 2, therefore heat radiation to the surroundings is not eliminated, depending on the used temperature sensor, where two states can occur. The first is prevailing heat transfer by radiation to the surroundings and negligible influence of the temperature sensor. The second is significant heat transfer via the sensor and its wires, this state is not desirable, because the measured temperature is decreasing. As previously mentioned, this state can influence the measurement by two ways, depending on the time-material parameters of the sensor. That means, the ratio between the warmed through mass of the sensor and the time of the accomplished measurement. If the mass of the sensor is considerable to the material of the measured object, a situation of a significant decrease of the measured temperature arises at the initial stage of measurement, and therefore, a negative error of a higher-order of transition characteristics. After a certain amount of time, which is determined by the intensity of the supplied energy, warm through of the sensor takes place [8], and it will behave as an object that inhibits the heat transfer, and, on the contrary, it increases the measured temperature. This is particularly evident in a steady state measuring [9].

After implementation of mathematical analysis it is necessary to create differential equations of the object and the measured circuit. In this case we can proceed with an algebraic approach, for the simpler systems. For more complicated systems it is convenient to use a mathematical apparatus, for example program SNAP.

Transmission with a radiating resistor R_0, is first-order with a time constant. By analysing the whole circuit we do not change the order of the system, we change only the ratio of the time constant T and output in a steady state (15-17).

$$G(s) = \frac{R_0}{(R_v \cdot C_m \cdot R_0)s + R_v + R_0}, \quad (15)$$

$$G(s) = \frac{\dfrac{R_0}{R_v + R_0}}{\dfrac{R_v C_m R_0}{R_v + R_0}s + 1}, \quad (16)$$

$$G(s) = \frac{K}{Ts + 1}, \quad (17)$$

where K – static gain;
T – time constant of transfer, s.

In Fig. 5 a model that is expanded by another structure, which determines transfer of the surface structure of the object, is described. [10]

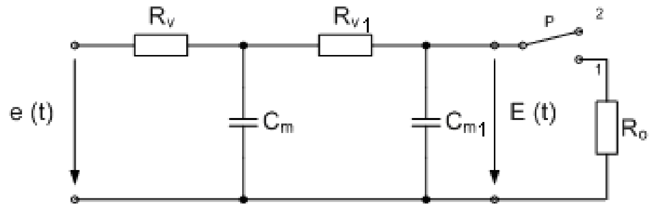

Fig. 5. **Model of the higher order object**

For coincident parameters of the linear netting structure RC ($R_v = R_{v1}$ and $C_m = C_{m1}$), the transfer is determined as (18-20).

$$G(s) = \frac{R_0}{(R_v^2 \cdot C_m^2 \cdot R_0)s^2 + (R_v^2 \cdot C_m + 3 \cdot R_v \cdot C_m \cdot R_0)s + 2 \cdot R_v + R_0}, \quad (18)$$

$$(R_v^2 \cdot C_m^2 \cdot R_0)s^2 \cdot Y(s) + (R_v^2 \cdot C_m + 3 \cdot R_v \cdot C_m \cdot R_0)s \cdot Y(s) + (2 \cdot R_v + R_0)Y(s) = R_0 U(s) . \quad (19)$$

$$R_v^2 \cdot C_m^2 \cdot R_0 \cdot y(t) + (R_v^2 \cdot C_m + 3 \cdot R_v \cdot C_m \cdot R_0)\int y(t)dt + (2 \cdot R_v + R_0)\int\int y(t)dt = R_0 \int\int u(t)dt . \quad (20)$$

In a general structure this dependence is valid, but in our case we have to modify the parameters R_v, C_m and R_{v1}, C_{m1}. Conversion to an integral form is necessary for modelling needs in MATLAB program, since the integral realisation is easy (21-23):

$$G(s)=\frac{R_o}{(R_v\cdot C_m\cdot R_0\cdot R_{v1}\cdot C_{m1})s^2+ +(R_v\cdot C_m\cdot R_{v1}+R_v\cdot C_m\cdot R_0+R_v\cdot C_{m1}\cdot R_0+R_{v1}\cdot C_{m1}\cdot R_0)s+R_{v1}+R_0+R_v}, \quad (21)$$

$$G(s)=\frac{\dfrac{R_0}{R_{v1}+R_0+R_v}}{\left(\dfrac{R_v\cdot C_m\cdot R_0\cdot R_{v1}\cdot C_{m1}}{R_{v1}+R_0+R_v}\right)s^2+ +\left(\dfrac{R_v\cdot C_m\cdot R_{v1}+R_v\cdot C_m\cdot R_0+R_v\cdot C_{m1}\cdot R_0+R_{v1}\cdot C_{m1}\cdot R_0}{R_{v1}+R_0+R_v}\right)s+1}, \quad (22)$$

$$G(s)=\frac{K_p}{a_{2p}s^2+a_{1p}s+1}, \quad (23)$$

where K_p – static gain.

If there is a need to create a 3-D model, it is necessary to approach an analogue of the 4th order abstraction model. This does not maintain details between sub-elements and the model. It uses the finite element network method, which consists of a system with distributed parameters, which is subsequently modelled by the system with lumped parameters [7].

Fig. 6 shows a string of basic design, the simple RC element as a heat source and the measuring system again presented only with the RC inertia where the resistor represents radiation into the surroundings [11; 12].

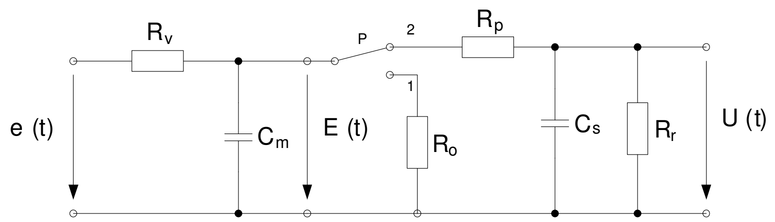

Fig. 6. **Model of the thermocouple circuit with the measured object in basic design**

Another technique will be addressed in MATLAB program. Fig. 7 shows an illustration of a model system that consists of a resistor (block RC point), a block representing transient effects (block g2), the sensor (block thermocouple) and the measuring system (block g4).

Resistor is in basic design with second-order inertia. Another block is a block representing a transient effect when the sensor is in contact with the object, which is stated as a first-order system. The actual sensor is provided with the first-order transmission with a time constant of 0.15 s. The time constant is determined by measuring the thermocouple. Concluding transmission element represents the response of the measuring system [13]. Individual transfers are determined by detection of the reverse transmissions in the chain, detection of known characteristics of the time constants declared by the manufacturer or established in the model. Transmission of the whole system is shown (24):

$$G_1(s)=\frac{y(s)}{u(s)}=\frac{1.705}{3\cdot10^{-9}s^5+6.29\cdot10^{-6}s^4+3.582\cdot10^{-3}s^3+0.2937s^2+1.846s+0.3}. \quad (24)$$

Fig. 7 shows a model in MATLAB program, the first part consists of a model pulse source of the signal, the second part shows the actual model of the objects and the last part shows a circuit for data recording. It is possible to set up a load-factor z and the number of cycles n_p. For analytical purposes, as in other parts, parameters are chosen of pulse duration t_1 = 20, 30, 40 ms; time t_2 = 500 ms and number of pulses $n_p = 60$. Number of cycles n_p assuming stabilization of the mean value of the measured temperature [14].

Furthermore, cases of model situations are included that are similar, for comparison, with the experimental part. The load-factor is determined as the ratio $\frac{t_1}{t_1 + t_2} = \frac{t_1}{T}$, where t_1 (s) is a pulse duration and T (s) the period of the signal.

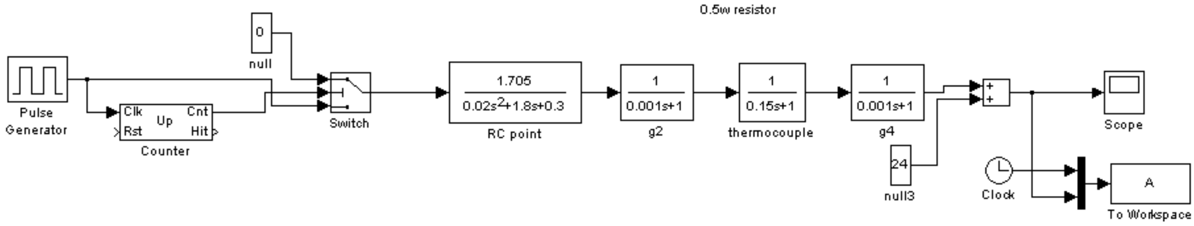

Fig. 7. **Scheme of the MATLAB program case**

Fig. 8 and Fig. 9 show experiment waveforms temperature from thermocouples with wire diameter of 0.012 mm and 0.12 mm. Fig. 8 shows a model temperature waveform with a load-factor $z = 0.0385$. Fig. 9 shows the temperature waveform when the load-factor was changed to a value $z = 0.0566$. The experiment is based on the pulse test, therefore, on the response of the system to the specified number of pulses to steady state temperature.

Apart from the time constant of the sensor, the temperature difference is also influenced by the connection of the thermocouple wires. Pulsing of the temperature pattern is clearly visible for the thermocouple CHAL -0005 where the actual temperature of the object is presented.

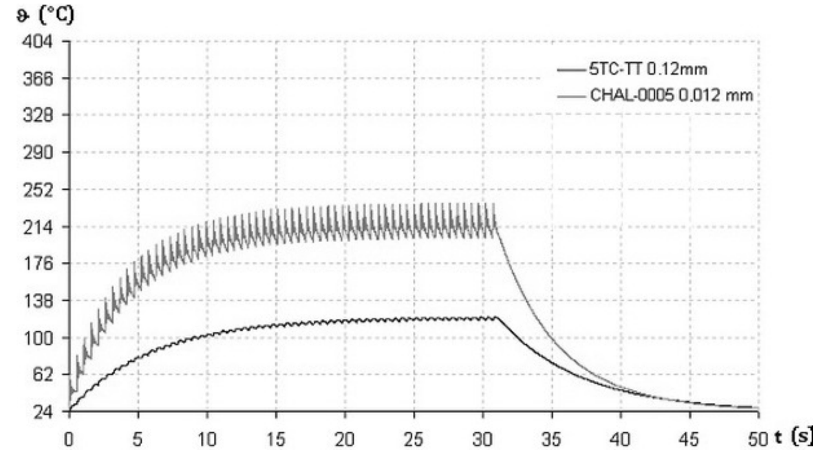

Fig. 8. **Model temperature waveform:** $n_p = 60$, $t_1 = 20$ ms, $t_2 = 500$ ms, $z = 0.0385$

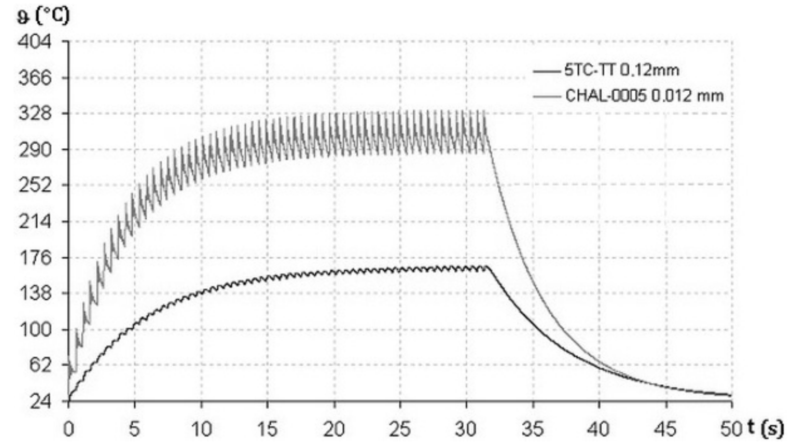

Fig. 9. **Model temperature waveform:** $n_p = 60$, $t_1 = 30$ ms, $t_2 = 500$ ms, $z = 0.0566$

Conclusions

The paper shows a possibility to model the temperature waveform of the resistance component dynamic loading. The model is implemented for the specific parameters of the measuring chain. Specifically, the model is implemented with a second order model of the measured component, the solution is based on a model of the object, and on the surface structure of the object. It is realized in MATLAB program. The analysis illustration is performed for the pulse test, i.e. the system response to the sequence of defined pulses. Modelling contributes to the possibility of object analysis without using real equipment, especially in situations where the device is operated on limit of its load bearing capacity. A dependence of the difference in the object temperature measurement by sensors with different time constants in area of rapid processes is shown. This case is important for pulse loading of the endpoint power stages where damage or change of the component parameters may occur during operation on the limit load of the objects. The created model enables us to conduct different variants of dynamic tests, especially in context of two types of testing, namely the pulse test and the test in a steady state that differ in a number of pulses and the power load factor.

The influence of the used sensor with a thermocouple wire diameter of 0.012 mm and 0.12 mm is evident on presentation of the actual temperature of the analyzed object. Mean temperature in steady state is 220 °C and 119 °C for the first case, and 309 °C and 165 °C for the second case. This difference is not negligible and the actual modeling was conducted on the limit of the component lifetime, with the provison that the resistive layer of the resistor is being already damaged.

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