DOUBLE PENDULUM MOTION ANALYSIS IN VARIABLE FLUID FLOW

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Abstract. The paper is devoted to the double pendulum motion analysis in the continuous media – fluid (wind, water) flow. Double pendulum is designed as two plates: the first plate moves around a fixed horizontal or vertical axis and the second plate is attached to the first by parallel axis. Continuous media (fluid) flow rate is selected such that can be described by a Fourier series first members. The first part of the article deals with the case when the pendulum moves in a vertical plane with horizontal axes and is subjected to the fluid interaction forces and gravity forces. In this article the fluid interaction forces are taken dependent on the flow velocity rate in square or linear. The motion differential equations of a system with two degrees of freedom are prepared. Analysis of these equations is made by computer. In the second part of the article the motion of the pendulum with vertical axis is analyzed. Additionally, the case when inside the pendulum axes elastic reversible springs are placed is recommended. The main motion parameter graphs are shown in transition and stationary vibration cases. Experimental investigations with the flow of air in the wind tunnel are mentioned. It is shown that the obtained results are useful for energy extraction from the fluid (air, water) flows.

Keywords: double pendulum, air flow, water flow, vibrations.

Introduction

This paper seeks to study how to use interaction of the fluid flow with the double pendulum system to generate stable stationary vibrations when the stream flow velocity is changing in time [1; 2].

Two models of the system (as two plates with length L1, L2 and width B) are presented in Fig. 1 and Fig. 2. In the first model the plates rotate around horizontal axes *z*1 and *z*2 with angular velocities *ω*1 and *ω*2 (Fig. 1.). Positions of the plates can be expressed by angles *φ*1 and *φ*2. Gravitational interactions G1 and G2 are in a vertical direction, parallel to Oy axis. Horizontal fluid flow velocity *V* positive direction is parallel to Ox axis. It is shown that for horizontal displacement of rotation axes the steering system for system orientation against the flow is necessary. Additionally, a torsional spring may be added in the case with vertical axes.

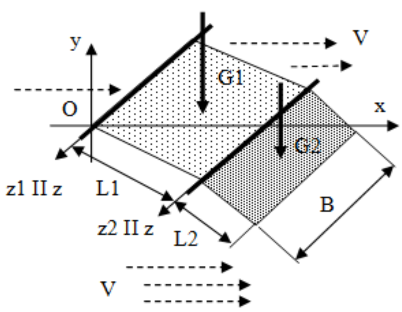

Fig. 1. **Model of pendulum with horizontal axes**

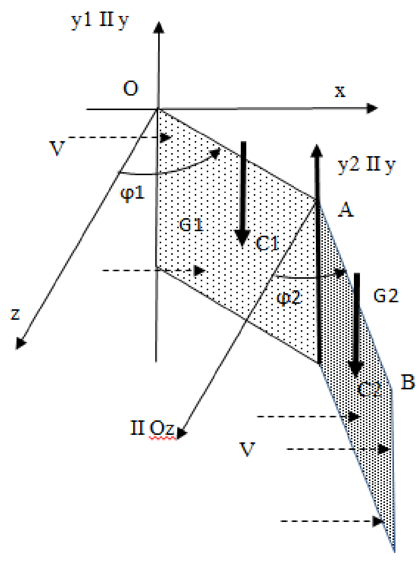

Fig. 2. **Model of pendulum with vertical axes**

Fluid flow interaction calculations

In this article flow interaction with plates as square dependency of relative velocity components is used (Fig. 3).

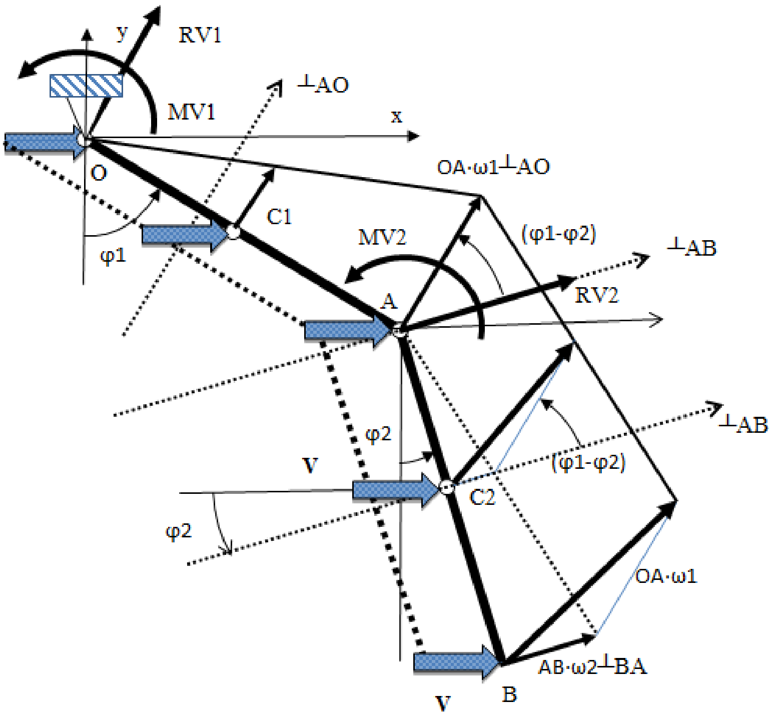

Fig. 3. **Scheme of fluid flow interaction forces simplifications:** *RV*1, *MV*1 – main vector and main moment of interaction forces of the first plate at axis Oz at point O; *RV*2, *MV*2 – main vector and main moment of interaction forces of the second plate in axis z2 at point A

For the first and second plate the main vectors *RV*1, *RV*2 and main moments *MV*1, *MV*2 of fluid flow interactions at points O and A are expressed by formulas (1), (2) [3]:

$$RV1 = D1 \cdot \int_0^{L1} (V \cdot \cos(\varphi 1) - \omega 1 \cdot \xi 1)^2 \cdot sign(V \cdot \cos(\varphi 1) - \omega 1 \cdot \xi 1) \cdot d\xi 1;$$
$$MV1 = D1 \cdot \int_0^{L1} (V \cdot \cos(\varphi 1) - \omega 1 \cdot \xi 1)^2 \cdot sign(V \cdot \cos(\varphi 1) - \omega 1 \cdot \xi 1) \cdot \xi 1 \cdot d\xi 1. \tag{1}$$

$$RV2 = D1 \cdot \int_0^{L1} [V \cdot \cos(\varphi 1) - \omega 1 \cdot \xi 1]^2 \cdot sign[V \cdot \cos(\varphi 1) - \omega 1 \cdot \xi 1] \cdot d\xi 1;$$
$$MV2 = D2 \cdot \int_0^{L2} [V \cdot \cos(\varphi 2) - \omega 1 \cdot L1 \cdot \cos(\varphi 1 - \varphi 2) - \omega 2 \cdot \xi 2]^2 \times \times \{sign[V \cdot \cos(\varphi 2) - \omega 1 \cdot L1 \cdot \cos(\varphi 1 - \varphi 2) - \omega 2 \cdot \xi 2]\} \cdot \xi 2 \cdot d\xi 2 \tag{2}$$

In the case when sub-integral functions in (1) and (2) are positive on bounds $\xi 1 = L1$ and $\xi 2 = L2$, e.g.:

$$V \cdot \cos(\varphi 1) - \omega 1 \cdot L1 \geq 0, \tag{3}$$

$$V \cdot \cos(\varphi 2) - \omega 1 \cdot L1 \cdot \cos(\varphi 1 - \varphi 2) - \omega 2 \cdot L2 \geq 0. \tag{4}$$

From formulas (1) and (2) can be found:

$$RV1 = D1 \left(\frac{L1^3 \cdot \dot{\varphi}1^2}{3} - L1^2 \cdot V \cdot \dot{\varphi}1 \cdot \cos(\varphi 1) + L1 \cdot V^2 \cdot [\cos(\varphi 1)]^2 \right), \tag{5}$$

$$MV1 = D1 \left(\frac{L1^4 \cdot \dot{\varphi}1^2}{4} - \frac{2 \cdot L1^3 \cdot V \cdot \dot{\varphi}1 \cdot \cos(\varphi 1)}{3} + \frac{L1^2 \cdot V^2 \cdot [\cos(\varphi 1)]^2}{2} \right), \tag{6}$$

$$RV2 = D2\begin{pmatrix} L2\cdot\left(V\cdot\cos(\varphi 2) - L1\cdot\dot{\varphi}1\cdot\cos(\varphi 1-\varphi 2)\right)^2 + \\ +\dfrac{L2^3\cdot\dot{\varphi}2^2}{3} - L2^2\cdot\dot{\varphi}2\cdot(V\cdot\cos(\varphi 2) - L1\cdot\dot{\varphi}1\cdot\cos(\varphi 1-\varphi 2)) \end{pmatrix}, \tag{7}$$

$$MV2 = D2\begin{pmatrix} \dfrac{L2^4\cdot\dot{\varphi}2^2}{4} + \dfrac{L2^2\cdot(V\cdot\cos(\varphi 2) - L1\cdot\dot{\varphi}1\cdot\cos(\varphi 1-\varphi 2)^2}{2} + \\ +[-\dfrac{2\cdot L2^3\cdot\dot{\varphi}2\cdot(V\cdot\cos(\varphi 2) - L1\cdot\dot{\varphi}1\cdot\cos(\varphi 1-\varphi 2))}{3}]. \end{pmatrix}. \tag{8}$$

The flow velocity function V in (1.5) – (1.8) can be expressed in Fourier series members as the function of time *t*:

$$V = V0\cdot(1+\lambda 1\cdot\sin(pt) + \lambda 2\cdot\sin(2pt+\alpha 2) + \lambda 3\cdot\sin(3pt+\alpha 3)+...), \tag{9}$$

where $V0,\ p,\ \lambda 1,\ \lambda 2,\ \alpha 2,\ \lambda 3,\ \alpha 3,\ \ ...$ – constants.

If inequalities (3), (4) are not fulfilled, the parameters *RV*1, *RV*2 and *MV*1, *MV*2 must be calculated only by numerical integration in time domain from integrals (1) and (2).

Linear dependency of interactions can be used for low speed vibration velocity and low fluid flow velocity, which gives the next equations:

$$\begin{aligned} RV1 &= D1\cdot[L1\cdot V\cdot\cos(\varphi 1) - \frac{L1^2\cdot\dot{\varphi}1}{2}]; \\ MV1 &= D1\cdot[\frac{L1^2\cdot V\cdot\cos(\varphi 1)}{2} - \frac{L1^3\cdot\dot{\varphi}1}{3}]; \end{aligned} \tag{10}$$

$$\begin{aligned} RV2 &= D2\cdot[L2\cdot V\cdot\cos(\varphi 2) - \frac{L2^2\cdot\dot{\varphi}2}{2} - L1\cdot L2\cdot\dot{\varphi}1\cdot\cos(\varphi 1-\varphi 2)]; \\ MV2 &= D2\cdot[\frac{L2^2\cdot V\cdot\cos(\varphi 2)}{2} - \frac{L2^3\cdot\dot{\varphi}2}{3} - \frac{L1\cdot L2^2\cdot\dot{\varphi}1\cdot\cos(\varphi 1-\varphi 2)}{2}]. \end{aligned} \tag{11}$$

Differential equations of motion

To find differential equations of motion for the system with two degrees of freedom the principle of virtual work was applied [4]. Virtual displacements $\delta\, rA\quad \delta\, rC2$ of points A and C2 can be expressed:

$$\delta\, rA = L1\cdot\delta\varphi 1;\quad \delta\, rC2 = r2\cdot\delta\varphi 2,$$

were $\delta\varphi 1, \delta\varphi 2$ – angular virtual displacements of the first and second plates.

Then, if $\delta\varphi 2 = 0$,but $\delta\varphi 1 \neq 0$:

$$\begin{aligned} &MV1 - M\Phi C1 - m1\cdot g\cdot r1\cdot\sin(\varphi 1) - \Phi C1Ot\cdot r1 + \\ &+ RV2\cdot L1\cdot\cos(\varphi 1-\varphi 2) - \Phi AOt\cdot L1 - \Phi C2At\cdot L1\cdot\cos(\varphi 1-\varphi 2) + \\ &+ (-\Phi C2An\cdot L1\cdot\sin(\varphi 1-\varphi 2)) - m2\cdot g\cdot L1\sin(\varphi 1)) - MG1 = 0. \end{aligned} \tag{12}$$

Additionally, if $\delta\varphi 1 = 0; \delta\varphi 2 \neq 0,$ then

$$\begin{aligned} &MV2 - M\Phi C2 - \Phi C2At\cdot r2 - \Phi AOt\cdot r2\cdot\cos(\varphi 1-\varphi 2) + \\ &+ \Phi AOn\cdot r2\cdot\sin(\varphi 1-\varphi 2) - m2\cdot g\cdot r2\cdot\sin(\varphi 2)) - MG2 = 0. \end{aligned} \tag{13}$$

Here

$$M\Phi C1 = J1\cdot\ddot{\varphi}1;\quad \Phi C1Ot = m1\cdot r1\cdot\ddot{\varphi}1;\quad \Phi AOt = m2\cdot L1\cdot\ddot{\varphi}1,$$

$$\Phi C2At = m2 \cdot r2 \cdot \ddot{\varphi}2; \quad \Phi C2An = m2 \cdot r2 \cdot (\dot{\varphi}2)^2,$$

$$M\Phi C2 = J2 \cdot \ddot{\varphi}2; \quad \Phi AOn = m2 \cdot L1 \cdot (\dot{\varphi}1)^2,$$

where g – free fall acceleration;
$J1$, $J2$ – moments of inertia pendulum parts;
$\dot{\varphi}1, \ddot{\varphi}2, \dot{\varphi}2, \ddot{\varphi}2$ – angular velocities and angular accelerations of pendulum parts with mases $m1$ and $m2$;
$MG1$, $MG2$ – damping forces moments like generators interactions.

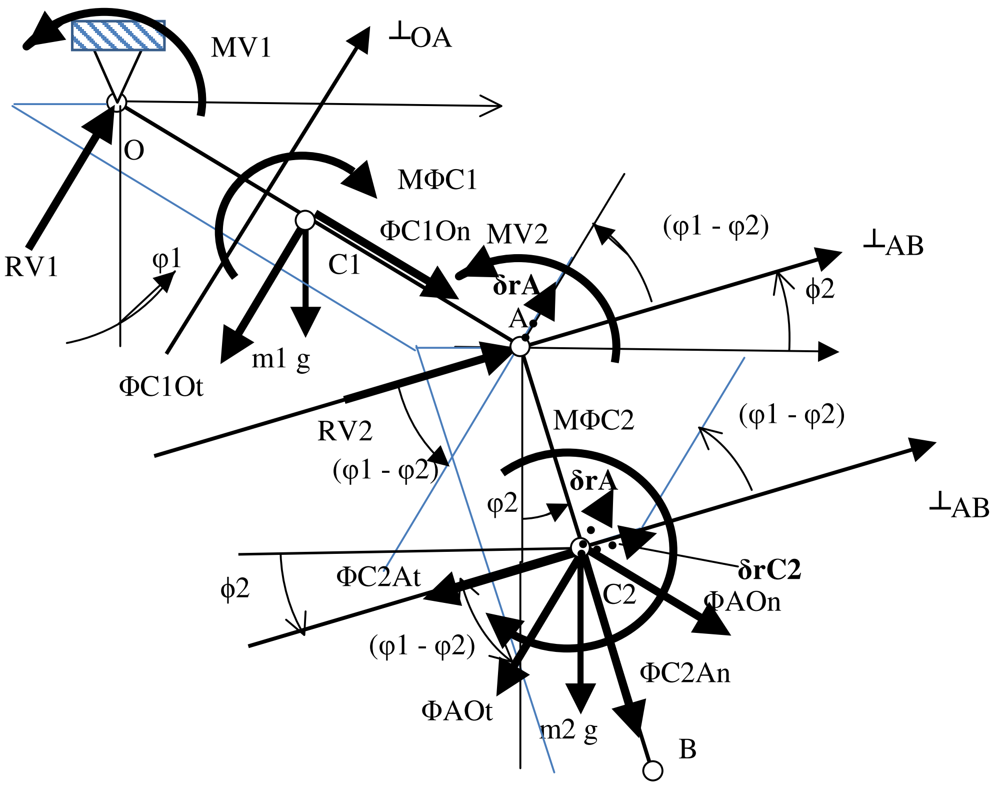

Fig. 4. **Scheme of forces for principle of virtual work**

After simplification of equations (12) and (13) it follows:

$$\begin{aligned} &MV1 - (J1 + m1 \cdot r1^2 + m2 \cdot L1^2) \cdot \ddot{\varphi}1 - m1 \cdot g \cdot r1 \cdot \sin(\varphi 1) + \\ &+ RV2 \cdot L1 \cdot \cos(\varphi 1 - \varphi 2) - m2 \cdot r2 \cdot \ddot{\varphi}2 \cdot L1 \cdot \cos(\varphi 1 - \varphi 2) + \\ &+ (-m2 \cdot r2 \cdot (\dot{\varphi}2)^2 \cdot L1 \cdot \sin(\varphi 1 - \varphi 2)) - m2 \cdot g \cdot L1 \sin(\varphi 1)) - MG1 = 0; \end{aligned} \quad (14)$$

$$\begin{aligned} &MV2 - (J2 + m2 \cdot r2) \cdot \ddot{\varphi}2 - m2 \cdot L1 \cdot \ddot{\varphi}1 \cdot r2 \cdot \cos(\varphi 1 - \varphi 2) + \\ &+ m2 \cdot (\dot{\varphi}1)^2 \cdot L1 \cdot r2 \cdot \sin(\varphi 1 - \varphi 2) - m2 \cdot g \cdot r2 \cdot \sin(\varphi 2)) - MG2 = 0. \end{aligned} \quad (15)$$

Example of modeling with MathCAD

Graphics of modeling equations (14), (15) in a case of linear interaction forces (10), (11) are shown in Fig. 5-7., with parameters $V0 = 5$ m·s^{-1}; $\lambda 1 = 0{,}2$; $\lambda 2 = 0.4$ and $\alpha 2 = 0$, see (9).

Comments about the motion characteristics are given under the graphics below.

Graphics of modeling of large flow velocity ($V0 = 15$ m·s^{-1}; $\lambda 1 = 0.2$; $\lambda 2 = 0.4$ and $\alpha 2 = 0$) in a case of linear interaction forces are shown in Fig. 8-10.

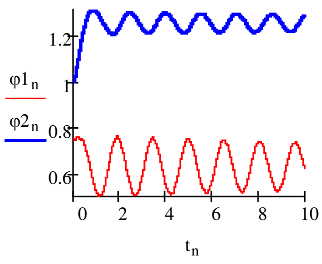

Fig. 5. **Graphics of rotation angles for plates.** Motion is very stable with low transition process

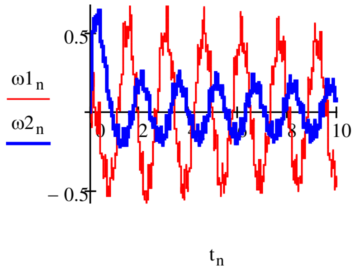

Fig. 6. **Graphics of angular velocities.** Amplitude of angular velocity of the first plate is two times more than for the second plate

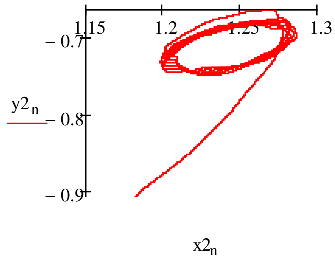

Fig. 7. **Motion of center mass C2 of second plate.** Trajectory is approximately ellipse

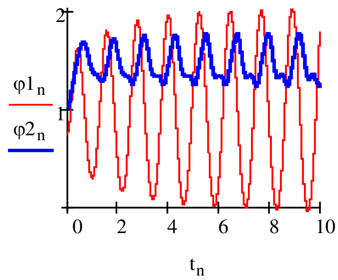

Fig. 8. **Graphics of rotation angles for plates with** $\boldsymbol{V0 = 15}$ **m·s**$^{-1}$**.** Motion is very stable with low transition process

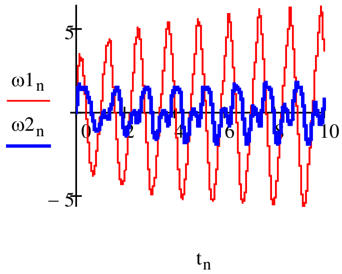

Fig. 9. **Graphics of angular velocities.** Amplitude of angular velocity of the first plate is three times more than for the second plate

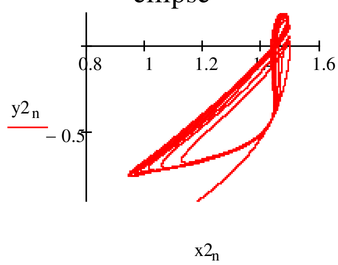

Fig. 10. **Motion of center mass C2 of second plate.** Trajectory is complicated

Example of modeling with Working Model 2D

Graphics of modeling double pendulum in air flow with different parameters with program Working Model are shown in Fig. 11-16. Comments about the motion characteristics are given under the graphics below.

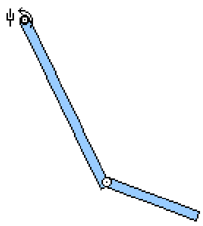

Fig. 11. **Model in vertical gravity interaction.** Air flow is horizontal. Graphics of motion angles are given in Fig. 12 and 13

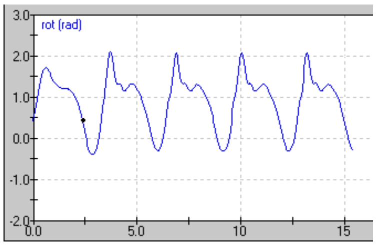

Fig. 12. **Graphics of first plate rotation angle with** $\boldsymbol{V0 = 10}$ **m·s**$^{-1}$ **and** $\boldsymbol{\lambda 1 = 0.2}$**;** $\boldsymbol{\lambda 2 = 0.4}$ **and** $\boldsymbol{\alpha 2 = 0}$

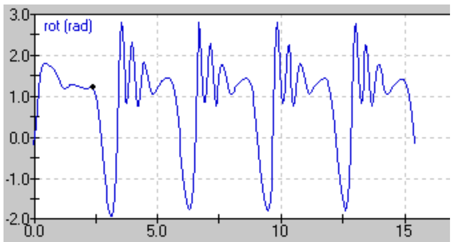

Fig. 13. **Graphics of second plate rotation angle.** Motion is very stable with low transition process

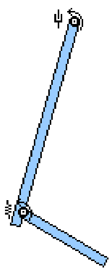

Fig. 14. **Model without vertical gravity interaction. Additional spring in the middle joint is added**

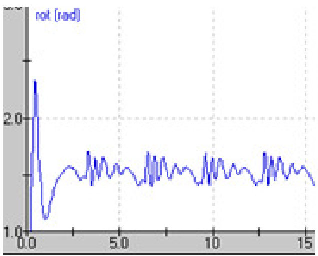

Fig. 15. **Graphics of first plate rotation angle with** $\boldsymbol{V0 = 10}$ **m·s**$^{-1}$ **and** $\boldsymbol{\lambda 1 = 0.2}$**;** $\boldsymbol{\lambda 2 = 0.4}$ **and** $\boldsymbol{\alpha 2 = 0}$

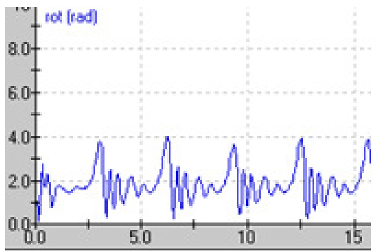

Fig. 16. **Graphics of second plate rotation angle.** Motion is very stable with low transition process

Results and discussion

The investigations show that for the vertical gravity model more efficiency is to add the energy generator to the first plate axis. For vertical mode of axes a small stiffness spring can be added to the middle joint axis. Today, the double pendulum model investigation problem with variable fluid flow for energy generations is in start position.

Conclusions

1. In real world the fluid flow (air or water) velocity is not constant but changes in time domain. These characteristics generate stable vibration motion in the double pendulum system.
2. The results of here investigated systems show that these vibrations can be used for energy extraction from the fluid flows.
3. Real applications of the offered devices need further investigations.
4. The two program (Math CAD and Working Model 2D) modeling results show a good coincidence with the theory.

References

1. Viba J., Vitols D., Tsyfansky S., Beresnevich V., Noskovs S., Jakushevich V. A vibrating object interaction with fluids. Scientific Journal of Riga Technical University. Transport and Engineering. Mechanics. Riga Technical University Press, 2015. pp. 7 – 11. (in Latvian).
2. Viba J., Eiduks M., Irbe M. Double pendulum vibration motion in fluid flow. International scientific conference "Engineering for rural development 2015". Jelgava, 20.-22.05.2015. pp. 434-439.
3. Anthony Bedford, Walace Fowler. Engineering Mechanics. Statics & Dynamics. Fourth Edition. Pearson Prentice Hall. 2005. - 622. p.
4. Goldstein H., Poole C., Safko J. Classical Mechanics, third edition, Addison-Wesley, 2002, 647 p.