SIMULATION OF ELASTIC DEFORMATION PROPAGATION OF GRAIN UNDER IMPACT CRUSHING IN CRUSHER

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Abstract. In order to raise its absorbency before feeding, or preparation of forage mixtures, it is necessary to crush or flatten the grain. A classic method of the grain preparation for feeding is its crushing by means of hammer crushers. There are presented the results of simulation of the propagation of the elastic and the plastic deformations in the individual grain as a plate of variable thickness under the impact of the hammer. The method of finite elements is used to compose a computation scheme taking into account the elastic and the plastic deformations, and stresses. There are the crushing zones of the grain defined at the collision with different initial velocities.

Keywords: grain, crusher, elastic deformation.

Introduction

At present concentrated fodder is widely used in the intensive stock farming on the basis of cereal crops. In order to raise its absorbency before feeding, or preparation of forage mixtures, it is necessary to crush or flatten the grain. A classic method of the grain preparation for feeding is its crushing by means of hammer crushers.

In the crushing chamber the material to be crushed is subject to repeated impact from the side of the operating tools of the crusher, changing the granulometric composition and shape of the circulating particles. The efficiency of the crushing process depends on: the way how the material is supplied to the hammer rotor, organisation of the air dispersion cycle inside the crushing chamber, the speed of the rotor hammers and withdrawal of the ready product from the crushing zone. The impact of these factors upon the crushing process of grain is the object of investigation by many scientists: V.P. Goryachkin, S.V. Melnikov, V.I. Syrovatka, etc. [1-3].

The process of impact crushing of individual grains using hammer crushers is accompanied by elastic and plastic deformations, which develop in a very short interval of time. In spite of the wide application of hammer crushers the issue concerning the theory of the propagation of the elastic and plastic deformations in the grain under impact has been studied insufficiently, there is no mathematical model of the disintegration process of the grain with respect to its shape, dimensions and elastoplastic properties. The purpose of this investigation is to develop a mathematical model of deformation propagation in an individual grain during impact crushing in the crusher.

Materials and methods

To solve the assigned task, the method of finite elements was used, applying it to the design diagram of the propagation process of the elastic deformation in a particle of the material to be crushed under impact, as well as detection of the transition moments into the plastic stage of deformation [4-6]. When building the model, we made an assumption that an individual grain (or its part) is a plate of variable thickness. The values of dimensions of separate regions of grains used in the numerical calculations were determined by means of a microscope. Collision of a grain with the hammer is companied by instantaneous constraints, which restrict the movement of the knots of the mechanical system. The initial relative velocities of all the points of the grain are the same and equal to the speed of the hammer in absolute motion (as this is a short-time collision, the movement of the hammer may be considered as translational). The knots which instantly stop, lose the degrees of freedom, therefore generalised movements $\{q\}$ corresponding to these knots are equal to zero. If from the matrix of rigidity and mass rows and columns corresponding to these generalised movements are crossed out, and from the vectors of the generalised coordinates and forces – the rows of these movements, then for the truncated system of equations we will obtain a matrix equation of free oscillations with positively defined matrices of masses $[M]$ and rigidities $[K]$ formed by joining the corresponding matrices of flat triangular finite elements of different width [5], i.e.:

$$[M]\{\ddot{q}\}+[K]\{q\}=\{0\}, \tag{1}$$

with the initial conditions

$$\{\dot{q}(0)\}=\{\dot{q}_0\}, \tag{2}$$

$$\{q(0)\}=\{0\}. \tag{3}$$

The algorithm of their formation is based on the topology of finite elements and represents a separate task to be solved in a numerical way in a packet of applied programmes [10].

The first part of the equation (1) are the generalised forces which depend on time and are equal to zero because they execute no constraint reactions applied at the point of contact, upon the possible generalised operation movements, and there are no other forces. The initial conditions for the generalised velocities (2) are defined as projections of the initial velocity of individual grain on axes x, y, i.e. all the even generalised velocities are equal to $\upsilon_y(0)$ but all the odd ones – to $\upsilon_x(0)$. Before the start of the account of the generalised movements we assume a condition of equilibrium, therefore the initial movements (3) are zero. At the first solution stage of equations (1) we will regard the coefficients of the matrix as constants, i.e. we suppose that the grain is an elastic body ignoring the plastic deformations. For positively defined matrices of masses and rigidities decomposition of the motion of the mechanical system is possible according to proper forms of oscillations. We will search a partial solution of the system (1) like:

$$\{q\}=\{A\}\sin pt. \tag{4}$$

where $\{A\}$ – a vector-column of amplitude values;

p – frequency of the proper oscillations of the mechanical system.

Substituting (4) into equation (1) after coefficient equalisation at function sin pt, we obtain:

$$\left[-p^2[M]+[K]\right]\{A\}=\{0\}. \tag{5}$$

A homogeneus matrix equation has not only a zero solution. In order to obtain non-zero solutions, it is necessary that the determiner of the matrix turned into zero. That is why there are proper values of the characateristic matrix $[K]^{-1}[M]$. These proper values p^{-2} determine the proper oscillation frequencies of the system described by the matrix differential equation (1) and the forms of its proper oscillations.The system of algebraic equations (5) has zero solutions $\{A\}$ only if the determiner of this system is equal to zero. System (5) can be reduced to the form:

$$[K]^{-1}[M]\{A\}=\frac{1}{p^2}[E]\{A\} \tag{6}$$

where $[E]$ is a unitary matrix of the same order as $[K]$ and $[M]$.

Then p^{-2} are the proper values of the matrix $[K]^{-1}[M]$ but the amplitude values $\{A\}$ are the proper vectors of this matrix. To compute the proper vectors $\{A\}_i$ and proper frequencies p_i, standard subprogrammes were used in the Fortran language: NROOT and EIGEN which operate simultaneously and are based on the Jacobi rotation method [7]. Index i is the number of the proper frequency of oscillations of the mechanical system $i = 1, 2\ldots, n$.

The forms of oscillations$\{A\}_i$, arranged in an ascending order of frequencies $p_1, \ldots, p_n$, constitute the matrix of the forms of oscillations:

$$[A]=\left[\{A\}_1\ldots\{A\}_n\right] \tag{7}$$

where n – the number of the freedom degrees of the mechanical system which is equal to the quantity of proper frequencies of oscillations.

The proper forms of oscillations [8; 9] have an orthogonality property; using it we transform equation (1) to the main coordinates. Multiplying it by the matrix $[A]^T$ and by the unitary matrix $[E]=[A][A]^{-1}$ we obtain:

$$[A]^T[M][A][A]^{-1}\{\ddot{q}\}+[A]^T[K][A][A]^{-1}\{q\}=\{0\} \tag{8}$$

The order of the matrices is determined by the quantity of knots when broken into finite (final) elements. The number of knots multiplied by 2 is equal to the dimension of the matrices *M* and *K*. In the given examples of estimates n = 102...104.

Let us mark:

- main matrix of masses as

$$[M_G]=[A]^T[M]\ [A]=[\ m_{Gi}\ ]=\begin{bmatrix} \diagdown & & \\ & m_{Gi} & \\ & & \diagdown \end{bmatrix};$$

- main matrix of rigidities [8] as

$$[K_G]=[A]^T[K]\ [A]=\begin{bmatrix} \diagdown & & \\ & k_{Gi} & \\ & & \diagdown \end{bmatrix};$$

- main generalised coordinates

$$\{u\}=[A]^{-1}\{q\};$$

- main generalised accelerations

$$\{\ddot{u}\}=[A]^{-1}\{\ddot{q}\}.$$

Then system (8) assumes the appearance:

$$[M_G]\{\ddot{u}\}+[K_G]\{u\}=\{0\}. \tag{9}$$

As the main matrices of masses and rigidities are diagonal, the system (9) breaks into separate equations of the appearance:

$$m_{Gi}\ddot{u}_i+k_{Gi}u_i=0\ . \tag{10}$$

The solutions of which we write down as:

$$u_i=C_{1i}\cos p_it+C_{2i}\sin p_it\ , \tag{11}$$

where $p_i=\sqrt{\dfrac{k_{Gi}}{m_{Gi}}}$ are proper frequencies of oscillations of the system.

Let us define the arbitrary constant of integration

C_{1i}, C_{2i} by initial conditions (2), (3). $C_{1i}=0$; $C_{2i}=\begin{bmatrix}\dfrac{1}{p_i}\end{bmatrix}\ [A]^{-1}\{\dot{q}_0\}$.

After this, returning to the initial coordinates, we obtain a solution of differential equations of the movement of the mechanical system discussed:

$$\{q\}=[A]\begin{bmatrix} \diagdown & & \\ & \sin p_it & \\ & & \diagdown \end{bmatrix}\begin{bmatrix}\dfrac{1}{p_i}\end{bmatrix}\ [A]^{-1}\{\dot{q}_0\}, \tag{12}$$

where the diagonal matrix contains values, reverse to the proper frequencies, on the main diagonal.

Formula (12) indicates how nodal displacements change in time; by them the relative deformations are determined in expression (14). If checking the system (1), nontruncated matrices [*M*] and [*K*] were used in it, then in the first part opposite the constraint movements (i.e. in the knots that are in contact with the hammer) we will obtain the values of reactions of the external links.

Results and discussion

A packet of applied programmes [10] has been worked out, implementing the described computing algorithm of the movement of the system and the interior tensions arising in the finite elements of the plate.

The intensity of stresses σ_u in a planar stressed state is computed according to formula [11]:

$$\sigma_u = \sqrt{\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2}\,, \quad (13)$$

where σ_x, σ_y, τ_{xy} – the components of stresses of the material in a planar stressed state, respectively, the normal stresses along the axes *x* and *y*, and the tangential stress τ_{xy}.

The intensity of the relative deformation ε_u [11]:

$$\varepsilon_u = \frac{\sqrt{2}}{3}\sqrt{(\varepsilon_y - \varepsilon_x)^2 + (\varepsilon_x - \varepsilon_z)^2 + (\varepsilon_y - \varepsilon_z)^2 + \frac{3}{2}\gamma_{xy}^2}\,, \quad (14)$$

is computed through ε_x, ε_y, γ_{xy} – deformation components of the finite element.

For the other elastic state a correlation called the generalised Hook's law is true [11]

$$\sigma_u = E\varepsilon_u\,.$$

In order to define the parameters of the movement of the system and internal stresses arising in the finite elements of the plate (the grain) and to obtain graphic dependencies on the basis of the model described above, a computation algorithm has been developed. The computation results of the stresses are presented in Fig. 1.

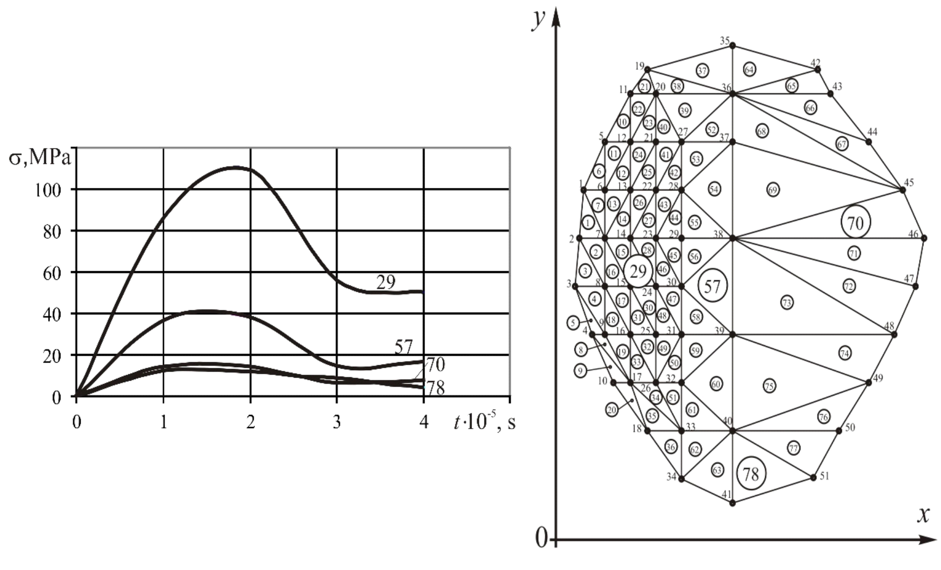


Fig. 1. **Dependence of stress σ on time for four elements of grain at the collision velocity v_0 = 75 m·s^{-1}, the elasticity module of the material *E* = 500 MPa (on the left the numbers of elements and knots of the grain are shown)**

The obtained solutions of the oscillation equations of the grain particles under impact without taking into consideration the elastic deformation produce overestimated values of stresses in the elements and a lesser than the real collision time [1]. Therefore, to raise the accuracy of the computation model, we will use an assumption of the elasticity theory [9]. Transition from the elastic state into a plastic state occurs when the value σ_u, called the intensity of stresses, reaches the yield limit σ_T. In the plastic region the link between the intensity of stresses and deformations can be presented more conveniently in the form of the generalised Hook's law:

$$\sigma_u = E_C \varepsilon_u , \tag{15}$$

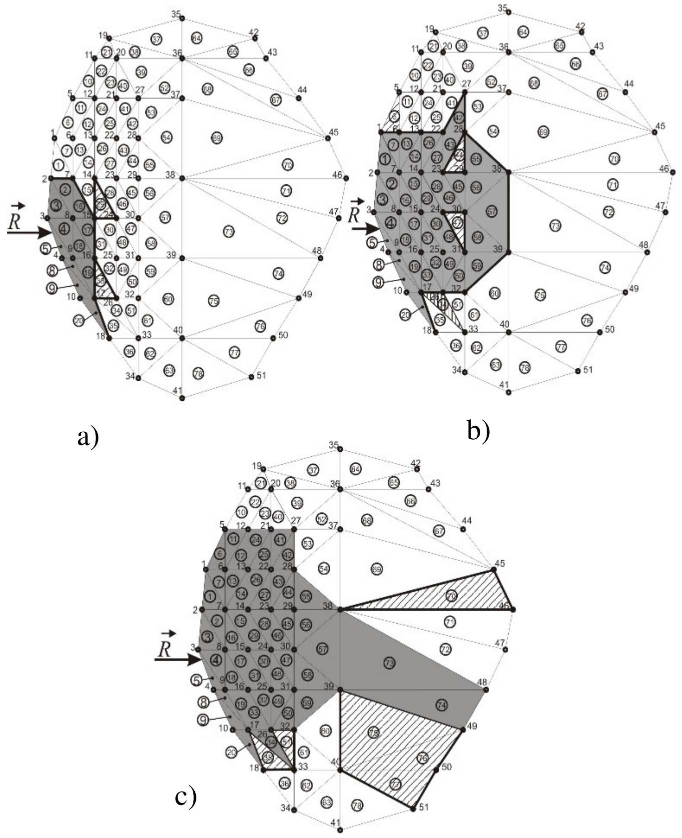
where E_C – regarded as the function of deformation ε_u and is called the secant modulus of elasticity $E_C = \dfrac{\sigma_u}{\varepsilon_u}$.

The secant modulus of elasticity is defined according to an experimental diagramme of a uniaxial stressed state for the current values σ_u and ε_u [9]. We will write the computational correlations for the deformation and stress components in the plastic region as:

$$\begin{cases} \varepsilon_x = \dfrac{1}{E}\left(\sigma_x - \mu\sigma_y\right) + \dfrac{3}{2E_n}\left(\sigma_x - \sigma\right); \\ \varepsilon_y = \dfrac{1}{E}\left(\sigma_y - \mu\sigma_x\right) + \dfrac{3}{2E_n}\left(\sigma_y - \sigma\right); \\ \gamma_{xy} = \dfrac{2(1+\mu)}{E}\tau_{xy} + \dfrac{3}{E_n}\tau_{xy}; \end{cases} \tag{16}$$

where $\sigma = \dfrac{\sigma_x + \sigma_y}{3}$; $\dfrac{1}{E_n} = \dfrac{1}{E_C} - \dfrac{2(1+\mu)}{3E}$.

For the discussed cases of the collision velocities there are zones of the individual grain built in Fig. 2 and Fig. 3, subject to plastic deformation $\sigma_u > \sigma_T$ (σ_T = 20 MPa), and a zone in which the intensity of stress reached the strength limit $\sigma_u > \sigma_B$, where σ_B = 27.6 MPa.



a) b) c)

Fig. 2. **Regions of maximal propagation of the plastic deformation ( ) and regions reaching the destructive deformations ( ):** a – υ_0 = 25 m·s^{-1} at the moment $t = 3.5\cdot10^{-5}$ s; b – υ_0 = 75 m·s^{-1} at the moment $t = 4.5\times10^{-5}$ s; c – υ_0 =125 m·s^{-1} at the moment $t = 5\cdot10^{-5}$ s

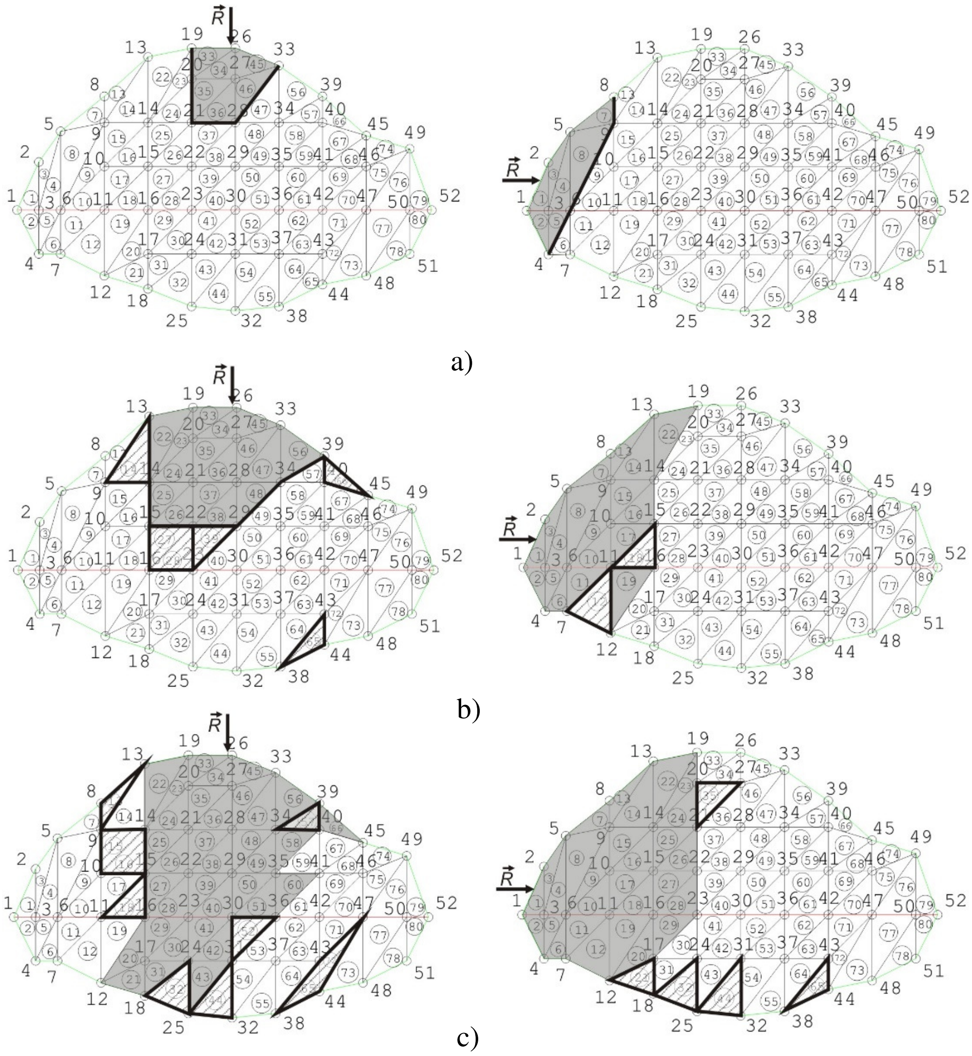


Fig. 3. **Scheme of the propagation areas of the plastic deformation (▨) and destructive deformation (■) at various impacts upon the grain:** a – $\upsilon_0 = 25$ m·s^{-1}; b – $\upsilon_0 = 75$ m·ss^{-1}; c – $\upsilon_0 = 125$ m·s^{-1}

In order to reach a positive effect from deeper crushing of grains in the plane of the minimal section, a scheme of a crusher is proposed withdrawing the grains from the crushing chamber along the tangent line towards the path of the hammers with a return in a perpendicular direction so that the grains, while sliding along the wall of the exit, are oriented in relation to the hammer in a plane with a lesser section [12].

Conclusions

1. On the basis of the method of finite elements there is produced a mathematical model of the propagation of the elastic and plastic deformations in the individual grain under the impact of the hammer as upon a plate of variable thickness.
2. The obtained deformation propagation regularities indicate that under the impact effect across the individual grain more efficient for crushing is the impact in the section of the lesser area, which can be explained by the bending deformation of the grain like a beam with a lesser moment of inertia of the cross section. In this case the time of collision is also slightly shorter. Under impact along the grain, propagation of deformation is approximately equal in both cross sections, which corresponds to the rod model of expansion-compression.
3. In order to reach a positive effect from deeper crushing of grains in the plane of the minimal section, a scheme of a crusher is proposed withdrawing the grains from the crushing chamber along the tangent line towards the path of the hammers with a return in a perpendicular direction

so that the grains, while sliding along the wall of the exit, are oriented in relation to the hammer in a plane with a lesser section.

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