CINEMATIC ANALYSIS OF PARTICLE OF IMPURITY IN CONDITIONING PROCESS OF GRAINS INTO INDENTED CYLINDER SEPARATORS

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Abstract. The paper has as a primary objective of mathematical modelling of the particles detachment from the alveoli of indented cylinder separators, highlighting the critical angle of detachment, position and initial velocity of the particle. The numerical simulation of the impurities detachment points and the particle trajectory after detachment from the alveoli, until collecting into the receiving gutter, has been performed using MathCad 12 software. The paper also has as a secondary objective of verifying the mathematical model by comparing the theoretical and experimental trajectory. In order to highlight the particle trajectory on experimental path, an indented cylinder separator stand, a high speed video camera and specialized software for advanced motion analysis have been used. The results have been obtained from experimental research which confirm the mathematical model.

Keywords: mathematical model, numerical simulation, indented cylinder separator.

Introduction

Before valorisation in various domains, grains are subjected to cleaning operations of foreign bodies. The grain conditioning operations for various destinations (sowing, grinding for consumption) are chosen depending on their type and quality, endowment with technical equipment, type of the product obtained, capacity of the plant etc.

The indented cylinder separators are technical equipment used within the grain conditioning technologies for sorting impurities contained in the grain mass (weed seeds, bindweed, fragments of wheat or other cereal grains that cannot be separated by sieving) using shape and length criteria.

Materials and methods

The mathematical modelling of particles detaching from the alveoli of the indented cylinder separator aims to obtain a sorting simulator with which the conditions for proper functioning of indented cylinder separators can be verified.

Table 1

List of mathematical model parameters

No.	Parameter-Name and significance	Notation	Unit
1	Gravitational acceleration	g	$m\cdot s^{-2}$
2	Cylinder radius	R	m
3	External friction angle of the main fraction of impurities on alveolus surface	ϕ	degree
4	Angular velocity of the cylinder	ω	$rad\cdot s^{-1}$
5	Rotational speed of the cylinder	n	rpm
6	Opening angle of the gutter	γ	degree
7	Angle between the normal to the alveolus surface and the normal to the cylinder surface at the detachment point	λ	degree
8	Mass of impurity particle	m	kg

The parameter significance can be better understood using Figure 1, obtained by adapting the information from [1].

According to [1 – 3], in case the cylinder has a horizontal axis, the critical angle of detachment is calculated based on the particle equilibrium equations at the point of detachment. The mobile reference system (solidary with the impurity particle), $\xi M\eta$ is centred in the point M, considered to be the point of detachment, $M\xi$, $M\eta$ being the tangent, respectively normal, in the point M, to the surface of the alveolus, in accordance with the representation from Figure 1. The equilibrium equations have the form:

$$\begin{cases} N' - mg\cos\left(\lambda - \alpha - \frac{\pi}{2}\right) + m\omega^2 R\cos(\pi - \lambda) = 0; \\ -m\omega^2 R\sin(\pi - \lambda) - F_f - mg\sin\left(\lambda - \alpha - \frac{\pi}{2}\right) = 0. \end{cases} \tag{1}$$

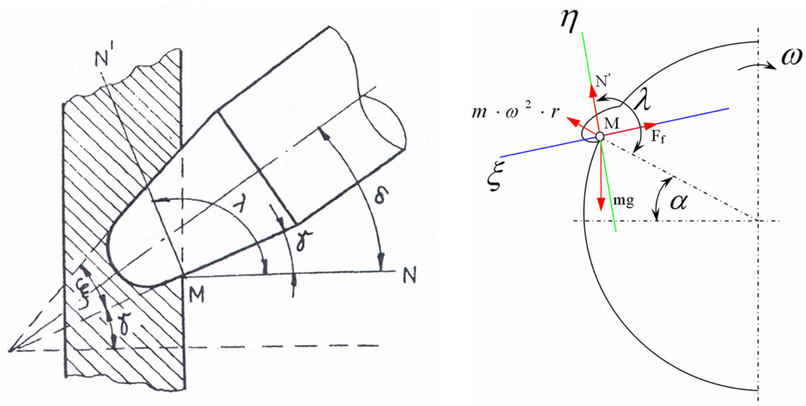

Fig. 1. **The shape of alveoli [1] and the system of forces acting upon the particles of impurities at the time of detachment from cylindrical surface**

By processing of the relationship (1) and considering the notation

$$k = \frac{\omega^2 R}{g} \tag{2}$$

where k is the index of kinematic regime, it obtains:

$$\alpha = \lambda + \phi - \arccos[k\sin(\lambda + \phi)] \tag{3}$$

By solving the problem of equilibrium the critical angle of detachment of the particle is obtained as the function of the main variables of the process:

$$\alpha(\lambda, \phi, \omega, R) = \lambda + \phi - \arccos\left[\frac{\omega^2 R}{g}\sin(\lambda + \phi)\right] \tag{4}$$

By knowing the constructive data of the cylinder and its rotational speed (λ, R, ω), as well as the variation interval of the external friction angle corresponding to the majority of the impurities that have to be removed, the equation (4) gives us the value of the critical angle of detachment of the particle.

By differentiating the equation (4) in the function of ϕ and using the notation (2), it obtains:

$$\frac{\partial\alpha}{\partial\varphi} = \frac{1 - k^2}{[\sqrt{1 - k^2\sin^2(\lambda + \varphi)} - k\cos(\lambda + \varphi)]\sqrt{1 - k^2\sin^2(\lambda + \varphi)}} \tag{5}$$

It is noticed that the denominator of the function (5) is positive. Therefore, if $0 < k \leq 1$, then $\frac{\partial\alpha}{\partial\phi} \geq 0$ and the function α is increasing with respect of ϕ. In order to determine the area covered by the particles that are detached and fall into the gutter, it is sufficient to determine the trajectories of the particles with minimum, respectively the maximum friction coefficient.

For the calculation of the trajectories of the detached particles, which are modelled as material points moving in vacuum, it is necessary to know the initial position vector and initial velocity vector, elements dependent on the variables of which the angle of detachment (λ, ϕ, ω, R) also depends.

The coordinates of the detachment point are calculated from the motion equations of the point *M* which moves on the circular trajectory of the radius R:

$$\begin{cases} Xd(\lambda,\phi,\omega,R) = R\cos[\pi - \alpha(\lambda,\phi,\omega,R)]; \\ Yd(\lambda,\phi,\omega,R) = R\sin[\pi - \alpha(\lambda,\phi,\omega,R)], \end{cases} \tag{6}$$

and the velocities are obtained by derivation the function of time:

$$\begin{cases} VXd(\lambda,\phi,\omega,R) = R\omega\sin[\pi - \alpha(\lambda,\phi,\omega,R)]; \\ VYd(\lambda,\phi,\omega,R) = -R\omega\cos[\pi - \alpha(\lambda,\phi,\omega,R)]. \end{cases} \tag{7}$$

The coordinates of the trajectories of impurities are obtained by integration of the motion equations, having the final form:

$$\begin{cases} X(\lambda,\phi,\omega,R,t) = Xd(\lambda,\phi,\omega,R) + VXd(\lambda,\phi,\omega,R)\cdot t; \\ Y(\lambda,\phi,\omega,R,t) = Yd(\lambda,\phi,\omega,R) + VYd(\lambda,\phi,\omega,R)\cdot t - \dfrac{g}{2}t^2. \end{cases} \tag{8}$$

As shown in relations (6), the position of the detachment point of one impurity depends on the angle λ characteristic to the alveolus profile which the cylindrical surface is endowed with, the friction angle ϕ between the impurity particle and the inner wall of the cylindrical surface (steel), angular velocity ω of the cylinder and its radius, *R*.

The mathematical modelling and the numerical simulation of the impurities detachment points and the particle trajectory after detachment from the alveoli, until collecting into the receiving gutter, have been performed using MathCad 12 software.

In order to verify the mathematical model which studies the kinematic parameters of conditioning into the indented cylinder separator, they have been highlighted experimentally (Figure 2), visually and numerically, the detachment angle of the alveolus, the trajectory, velocity and absolute acceleration of the impurity (or its axial components). For a particle of impurity selected from the many impurities passed in front of the optical objective, there were measured - the angle of detachment, trajectory, velocity and acceleration and then the correctness of the mathematical model that calculates these parameters was verified numerically. The system consisting of high-speed video camera, Phantom V10 (Vision Research), artificial light sources (4 projectors with aggregate power of aprx. 2000 W) and motion analysis software TEMA Automotive (Image System AB) enables filming in good condition with a frame rate of 1800 fps, which means that every 1/1800 s = 0.55 ms the system provides us the position, velocity and acceleration of the particle, with a good precision.

Fig. 2. **Aspects during experimental research using high speed video camera**

Results and discussion

If the primary objective of the paper was the mathematical modelling of the particles detachment from the alveoli of indented cylinder separators, highlighting the critical angle of detachment, position and initial velocity of the particle, the secondary objective was comparing of the results obtained theoretically and experimentally, for the kinematic parameters of the working process of the indented cylinder separator. The theoretical results are obtained based on the mathematical model developed before and numerically simulated in MathCad, and the experimental results are obtained based on using high-speed video equipment and the motion analysis software.

Using the motion analysis software (Figure 3), the moment has been followed when a particle of impurity left the alveolus and from that moment until the collection into the receiving gutter, the kinematic parameters of this particle were determined optically based on the shape analysis in the images stored by high speed video camera. Therefore, the motion analysis software has determined on the image the detachment angle of the alveolus, initial position and velocity of the impurity particle and has calculated the values of the velocity and acceleration vectors (absolute or axial component) along the trajectory until the collection gutter.

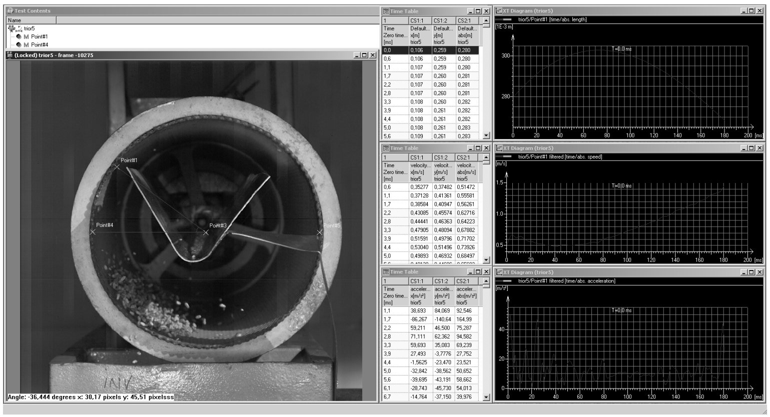

Fig. 3. **The graphical user interface of the motion analysis software**

The research purpose is to verify the correctness of the mathematical model.

The mathematical model results are influenced by the following simplifying hypotheses:

- the impurity is described as a material point, so with no spatial extent, no size, only the mass, which is wrong, because the impurities have small mass but have spatial extent and well defined stereometry characteristics (moments of inertia, etc.);
- due to the small mass of impurities and large spatial extent, the dynamic coefficients of impurities are high and these particles are braked or driven by air currents within the cylinder, which alter their trajectory;
- the impurities movement as material points, described in the literature, is made in vacuum, so air friction is not taken into consideration (a more realistic modelling could be obtained by considering the impurity as a material point which moves in a medium with friction).

There were represented the variations in time of displacement in horizontal projection, the variations of vertical components of displacement, displacement trajectories, the variations in time of horizontal velocities, the variations in time of velocity vertical projection, the variations in time of resultant velocities, the variations in time of horizontal acceleration and the variations in time of vertical acceleration. The overlapping of the corresponding components of the experimental movement (real) and theoretical movement (ideal) appears in Figures 4-11.

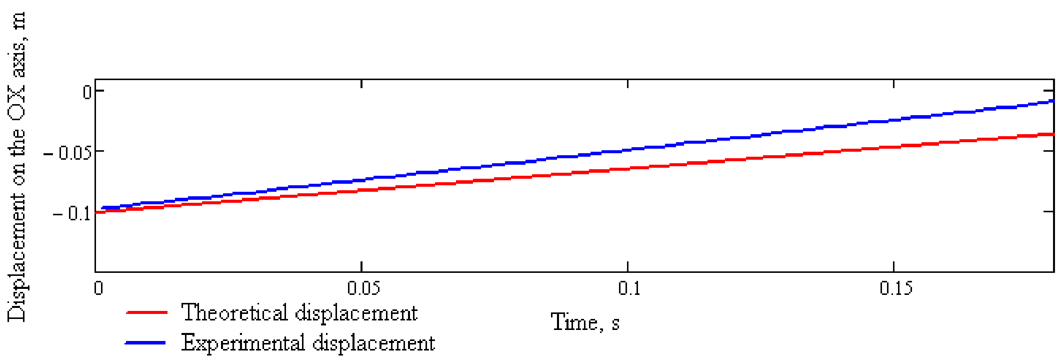

Fig. 4. **Variations in time of displacement in horizontal projection**

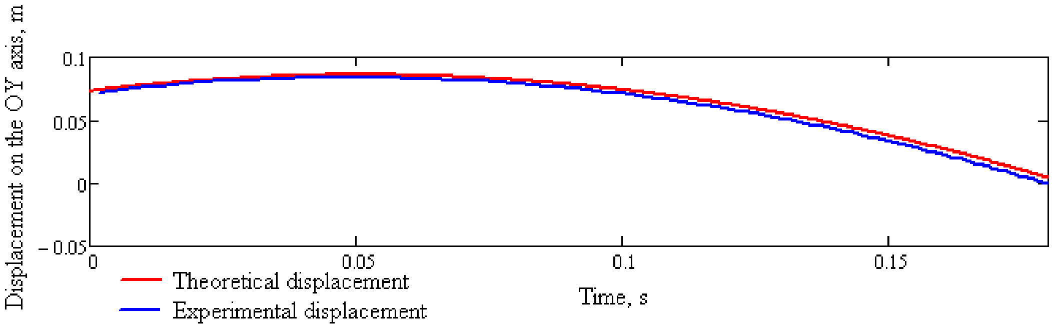

Fig. 5. **Variations of vertical components of displacement**

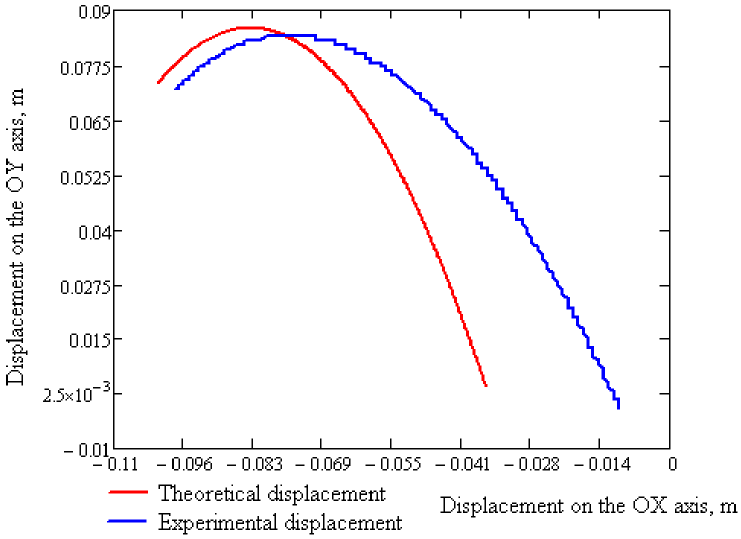

Fig. 6. **Displacement trajectories of the particles**

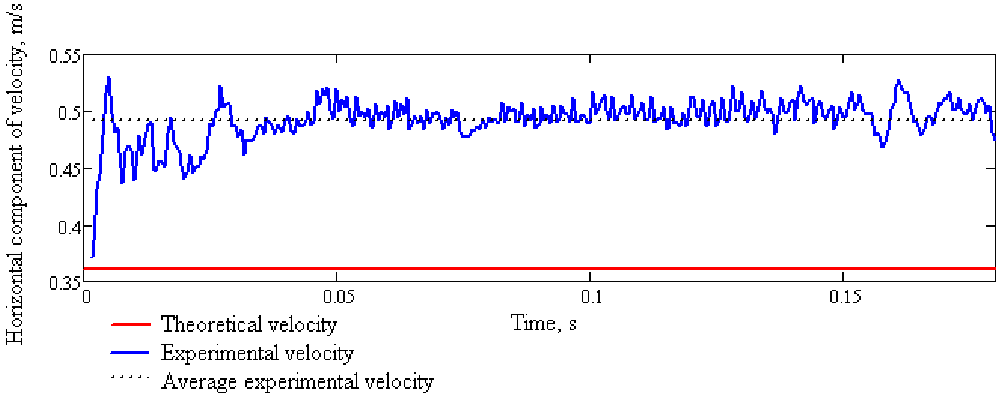

Fig. 7. **Variations in time of horizontal velocities**

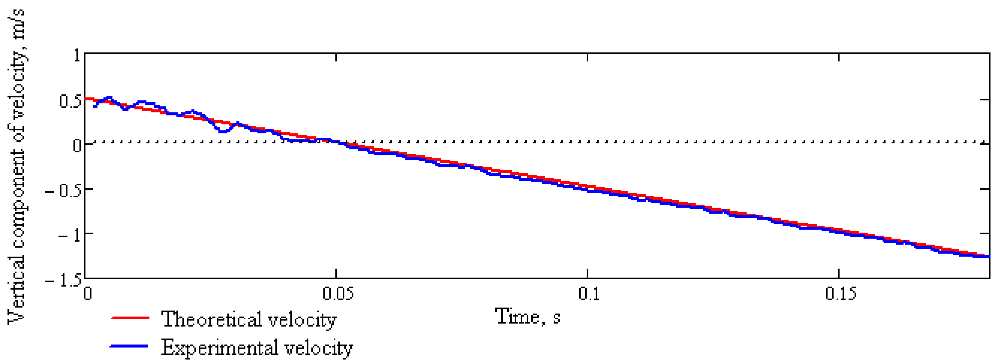

Fig. 8. **Variations in time of velocity vertical projection**

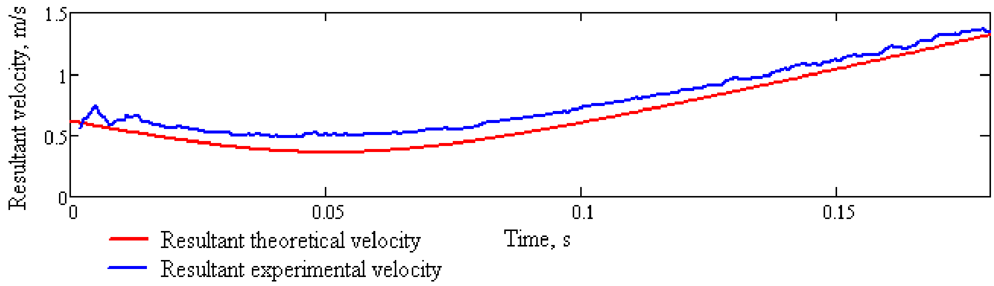

Fig. 9. **Variations in time of resultant velocities**

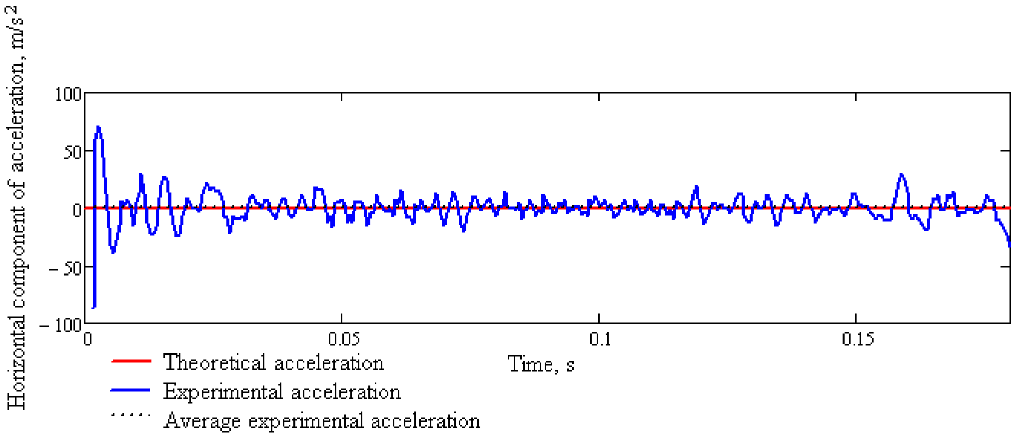

Fig. 10. **Variations in time of horizontal acceleration**

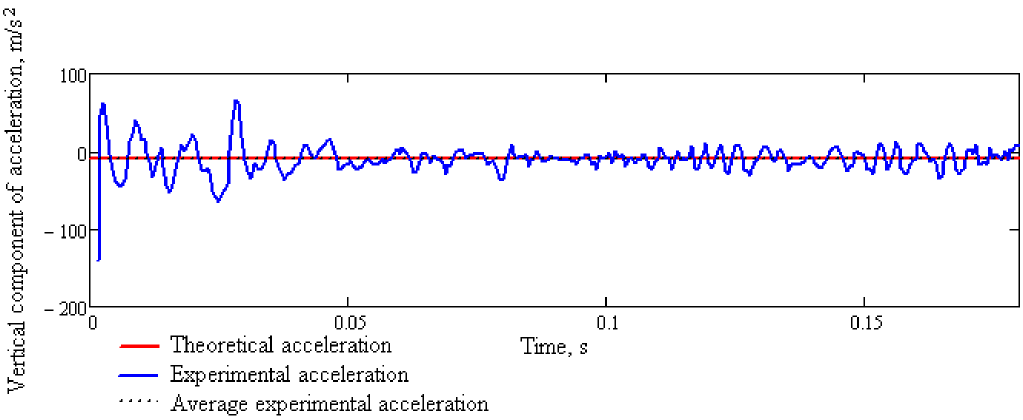

Fig. 11. **Variations in time of vertical acceleration**

Conclusions

After comparing the results obtained by theoretical and experimental means for the kinematic parameters of the indented cylinder separator working process, the following conclusions were highlighted:

1. In shape, the real movement is similar to the parabolic, ideal (obtained based on the mathematical model developed);
2. It was found that the motion of the particles differs more on horizontal, the trajectories being different especially after reaching its maximum height;
3. Vertical acceleration measured by filming is very close to the ideal gravitational acceleration;
4. Reducing the vertical acceleration (lower in the absolute value than the ideal one) is caused by the friction with air and the air currents possibly generated by the working process and, at high flow rates, by collisions with other particles, which makes the movement a random one;

5. The processing of images has recorded the particle not from the actual detachment point but after its complete identification (a proper operating condition of the shape recognition algorithm within the advanced image analysis software), the moment that the particle has already left the cylinder surface (with 3 mm);
6. The mathematical model developed and numerically simulated, approximates with sufficient precision the actual point of detachment of an impurity particle from the alveolus and its trajectory until it is collected into the receiving gutter;
7. By knowing the variation interval of the external friction angle corresponding to the majority of impurities that must be removed from the grain mass, in order to properly adjust the position of the receiving gutter depending on the area covered with impurity particles that detach and fall into the gutter, it is sufficient to determine the particles trajectories for the minimum and maximum friction coefficient.

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