IMPORTANCE OF THEORETICAL KNOWLEDGE IN MATHEMATICS STUDIES IN ENGINEERING PROGRAMS

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Abstract. Engineers use mathematics programs for solving engineering problems and development of these skills for students is important. But understanding what problems could be calculated by programs and being able to create and solve problems is not less important. Students are also required to have theoretical knowledge for solving problems. The aim of the article is to explore the possibilities of improving the knowledge of mathematics and to develop the necessary competences through changes in the mathematics study process and the use of e-learning. The results of the research show that dividing theory tests into 3 parts and writing them before the task test improve the students' task test result. The lecturers of the Department of Mathematics created interactive self-assessment theory tests using environment Moodle to improve the students' understanding of the mathematical laws and their ability to better prepare for the test. The research findings show that students, who had used the self-assessment theory tests in e-environment, improved the results of the theory tests by approximately 25 %, the results in the task tests by approximately 30 % and the results in the final exam by approximately 21 % in the analyzed period.

Keywords: Mathematics competences, study process in Mathematics, theoretical knowledge.

Introduction

Contemporary engineering education should be able to prepare specialists in accordance with the requirements of the era of information technologies and the labor market demands. **Universal and sustainable skills**, which will be useful for various areas of professional activity in the long-term period, are important both, for production and university education. [1].

Dublin Descriptors has been developed to outline the essential components of any degree programme. Qualifications that signify completion of the first cycle are awarded to students who [2]:

- have demonstrated knowledge and understanding in a field of study that builds upon and their general secondary education, and is typically at a level that, whilst supported by advanced textbooks, includes some aspects that will be informed by knowledge of the forefront of their field of study;
- can apply their knowledge and understanding in a manner that indicates a professional approach to their work or vocation, and have competences typically demonstrated through devising and sustaining arguments and solving problems within their field of study;
- have the ability to gather and interpret relevant data (usually within their field of study) to inform judgments that include reflection on relevant social, scientific or ethical issues;
- can communicate information, ideas, problems and solutions to both, specialist and non-specialist audiences;
- have developed those learning skills that are necessary for them to continue to undertake further study with a high degree of autonomy.

Mathematics is an integral part of engineering education, it is therefore essential to be aware of the role of mathematics in modern engineering education and the labor market. The European Society for Engineering Education (SEFI) in the latest edition of "Framework for Mathematics Curricula in Engineering Education" [3] states that although contents is still important, however, skills development of using and applying mathematical concepts in relevant contexts and situations certainly is a predominant goal of the mathematical education for engineers in the 21st century.

On the one hand, different mathematical IT programmes are applied in solving different mathematical problems. Due to this reason many stakeholders hold the view that students should not be "tortured" with complicated calculations in higher mathematics – it would be enough if they learn the main notions and use the relevant programmes in problem solution. On the other hand, the situation in the labor market is dynamic: new products are launched, production management system, equipment, technological processes etc. change persistently. Also the amount of scientific information is growing rapidly, therefore the university is not able to provide a student with the whole amount of the scientific knowledge and skills necessary for the whole working life. Therefore, engineering education is required for general courses such as mathematics, physics, etc., which develop cognitive skills and provide the fundamental knowledge and competences necessary for engineers.

The problem of the research is to study general competences developed by teaching of mathematics, to study importance of the theoretical knowledge in Mathematics Studies for providing the necessary competences and to explore the possibilities of improving the knowledge of mathematics through changes in the mathematics study process and the use of e-learning.

Materials and methods

Mathematical competence is an integral part of any engineering education. Almost all branches of engineering rely on mathematics as the language of description and analysis. Mathematical competences have a complicated structure. One of widely used concepts of mathematical competence was developed in Denmark and later adopted in the famous OECD PISA study [4]. M. Niss [5] stated, “Possessing mathematical competence means having knowledge of, understanding, doing and using mathematics and having a well-founded opinion about it, in a variety of situations and contexts where mathematics plays or can play a role.” The issue of mathematical competence has become one of the key education priorities on the EU policy level. According to Eurydice [6], ”in the past years, and especially since 2007, the great majority of European countries have revised their mathematics curricula, adopting an outcome-based approach whereby the focus lies on developing students’ competences and skills rather than on theoretical content”.

The substance of mathematics for universities is determined by the SEFI as follows [7].

1. Mathematics as a subject sees mathematics as part of the degree programme, to be studied via various teaching and learning techniques;
2. Mathematics as the basis of other subjects, both for study and in world at large, is something existing in its own right, something to be tackled (learned and understood) for future appropriate use;
3. Mathematics as a tool for analysing problems that occur in the world at large and hence solving them is co-existing with other areas of knowledge and supporting the study and development of that knowledge.

Kjeld B Laursen [8] cooperating with the Institute of Mathematical Sciences, Centre for Science Education, University of Copenhagen designed the eight mathematics competences (Fig. 1).

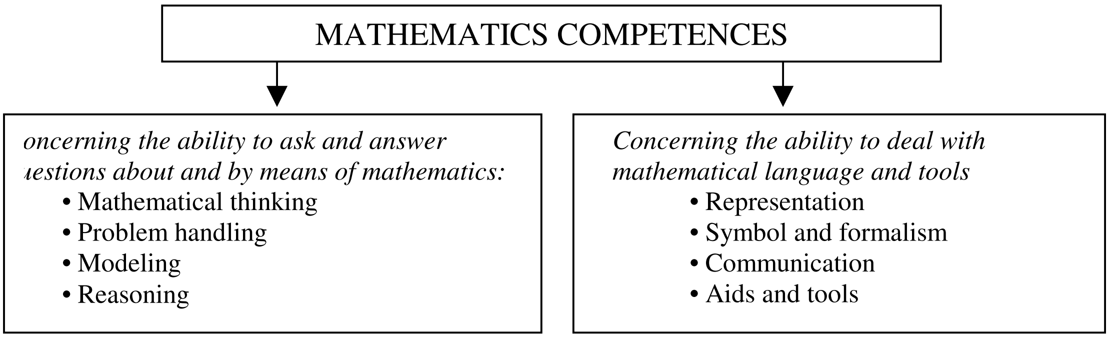

Fig. 1. **Mathematics competences**

Mathematical thinking competence means mastering mathematical modes of thought. It is competence of awareness of the types of questions that characterize mathematics, of ability to pose such questions and insight into the types of answers that can be expected.

Problem handling competence means being able to formulate and solve different kinds of mathematical problems, pure and applied, open and closed. It is competence to solve mathematical problems, if already formulated, whether posed by oneself or by others, and, if necessary or desirable, in different ways.

Modelling competence means being able to analyze and build mathematical models concerning other areas, i.e., able to: 1) analyze the foundations and properties of existing models and being able to assess their range and validity; 2) perform active modelling in a given context, i.e., mathematizing and applying it to situations beyond mathematics itself.

Reasoning competence means being able to reason mathematically: 1) the ability to follow and assess mathematical reasoning, i.e., a chain of arguments put forward by others, in writing or orally, in support of a claim; 2) the ability to produce and carry through informal and formal reasoning (based on intuition); including transforming heuristic arguments into formal, correct proofs.

Representation competence means being able to handle different representations of mathematical entities:1) understand (i.e., decode, interpret, distinguish between) and utilize different kinds of representations of mathematical objects, phenomena, problems or situations (including symbolic, algebraic, visual, geometric, graphic, diagrammatic, tabular or verbal representations, but also concrete representations of material objects); 2) understand the reciprocal relations between different representational forms of the same entity as well as knowing about their strengths and weaknesses, incl. loss or increase of information.

Symbols and formalism competence means being able to handle symbols and formal mathematical language, i.e., to: 1) decode symbol and formal language; 2) translate back and forth between mathematical symbol language and natural language; 3) treat and utilize symbolic statements and expressions, including formulas.

Communication competence means being able to communicate in, with and about mathematics, i.e., to: study and translate written, oral or visual mathematical expressions or texts, and express oneself in different ways and with different levels of theoretical or technical precision about mathematical matters, written, oral or visual, to different types of audiences.

Aids and tools competence means being able to make use of and relate to the aids and tools of mathematics (incl. IT), i.e.: 1) know about the existence and properties of relevant tools used in mathematics; 2) have insight into their possibilities and limitations; 3) be able to use such aids reflectively.

Theoretical knowledge in Mathematics studies is very important for providing the necessary competences. Firstly, it is the skill to apply formal rules exactly, which is sometimes abstract, complicated and multi-graded. An important skill is to choose exactly the rule which is necessary from a long list of known rules to complete the task, to design a solution plan or to design a sequence for applicable rules. Management skills are being developed here. Demonstrations of different theorems are significant in the Mathematics study process. Acquiring demonstrations, students develop skills of logical conclusions. Unfortunately, the number of lessons for Mathematics has decreased, and there is a tendency not to practice Mathematical demonstrations, but to teach students to calculate certain problems. Thus, the focus is on the applicability of Mathematics, not on Mathematics that develops engineering thinking with its formal logics.

Secondly, Mathematics is a language of symbols; it combines a continuous unity of the verbal expression and the sub-language of special symbols with exact rules. The problem definition is in the language, which does not use Mathematics concepts, symbols, axioms, theorems, formulae etc. Analysing solutions, the understanding should be focused on the symbols, letters, and signs in algebra. As a result the problem could be written in a shorter way than in the first version. Then we can perform algebraic modifications (make Transformation). Further we return from the language of symbols back to the usual language and create judgments.

Thirdly, long chains of logic conclusions are characteristic of Mathematics. If students have not practiced long chains of logical conclusions before the university, they are ready neither physically, nor mentally for the hard work to acquire long information units, therefore the studies at the university will be complicated, if not impossible. Dealing with reduced attrition (mental and physical) is one of the students' Mathematical ability components. Cognitive load may be reduced using a kind of "information quanta" – it is a designation for a portion of information with a new symbol. It is called a method of substitution in Mathematics.

Results and discussion

To improve Mathematics studies at the Latvia University of Agriculture (LUA), Department of Mathematics developed a structural scheme of study form modules and, according to them, the assessment system (Fig. 2). The study process contains four study forms – lectures, tasks, laboratory works, which are contact lessons, and students' individual work. E-environment is used in the study process and it contains materials on the theory, individual tasks, which are generated using Random Assignment Generator (RAG) from Moodle Question bank as well as Step by step examples of practical mathematics problem solutions with MathCad. Students solve individual works step-by-step and check the result by the program MathCad in computer classes. Submitted and passed individual works are required as a prerequisite to take the exam. During the semester students write a theory test, accounting for 8 % of the final assessment, and task tests accounting for 22 % of the final assessment. The final assessment consists of work during the semester (30 % of the assessment) and the final exam (70 % of the assessment).

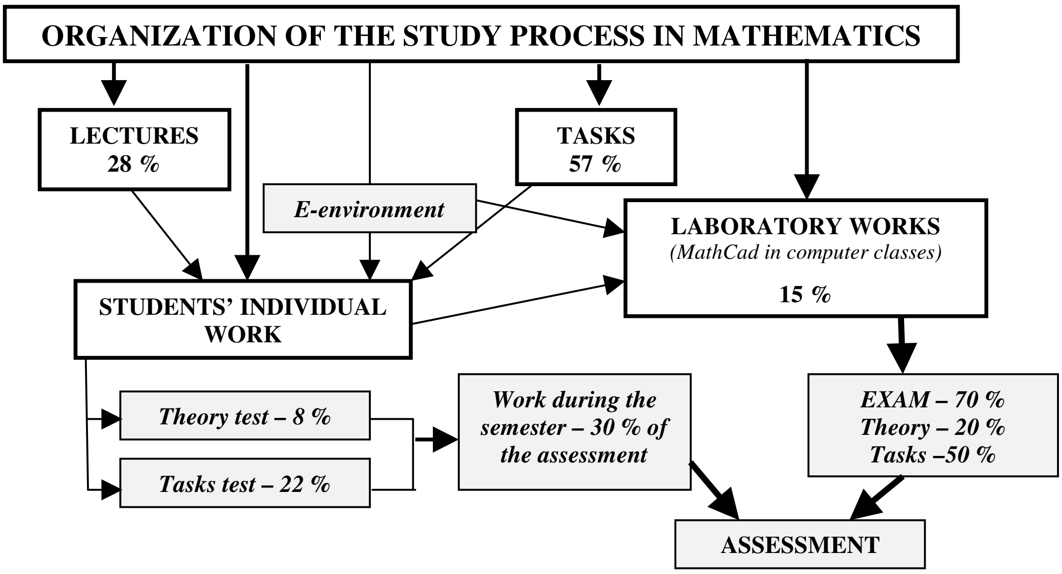

Fig. 2. **Structural scheme of study form modules and evaluation system in Mathematics at Latvia University of Agriculture**

Since the number of contact lessons in last years was small, students wrote three task tests and only one theory test during the semester. The theory test was written at the end of the semester. At the beginning of the research the results showed that the theory test marks were very low. The preparedness to the theory test could not help get better results in the three task tests. On the other hand, as the theory test was written at the end of the semester, students had a possibility to better prepare for the final exam. They could understand that it is necessary to learn more. Unfortunately, the result of the final exam was not so good.

The lecturers of the Department of Mathematics at the LUA have been looking for ways of changing tasks and using e-environment to give students a possibility to get more knowledge (theoretical and practical) in Mathematics before the task tests or the final exam. Since 2011/2012 study year, it was decided to divide the theory test into three parts. Each part is 15 minutes long and it is written before the task test. Scheduling the theory test before the task test makes students understand better methods and logical conclusions in the respective theme. Students become aware of not enough knowledge as well. Before the task test students attend lecturers' consultations to get deeper knowledge in Mathematics.

Comparing the study results in mathematics of the students of Faculty of Information Technology during the last four years, it is possible to conclude that in the last two years the theory and task test results were better when students wrote three parts of the theory test during the semester, as it can be seen from Table 1. Also the final exam grades were higher in 2011/2012 and 2012/2013 study years. In 2009/2010 and 2010/2011 study years the average assessment grades of the theory test were 4.3 and 4.1, respectively, when the theory test was written only once. In the next two years the assessment grades were 5.5 and 4.9, when the theory test was divided into three parts. Accordingly, the results of the theory test improved by about 23 %. Considering the results of the task tests in the first two years of the analyzed period, the assessment grades were 3.1 and 2.8; in the next two years the results were better: 4.6 and 3.6, respectively. In the last two years students were prepared for the task tests more seriously because they learned theory before the tests. Accordingly, the results of task tests improved by about 38 %. Also the final exam grades were higher by approximately 23 % in 2011/2012 and 2012/2013 study years as in the first two study years of the analyzed period.

Table 1

Study results of theory tests, task tests and the final exam in Mathematics of students from the Faculty of Information Technology

Study years	**Theory test**			**Tasks test**			**Exam**		
	Mean	*Me*	*Mo*	*Mean*	*Me*	*Mo*	*Mean*	*Me*	*Mo*
2009/2010	4.3	4.5	4	3.1	3.0	3	4.7	5.0	5
2010/2011	4.1	4.0	4	2.8	3.0	3	4.6	4.5	4
2011/2012	5.5	5.0	5	4.6	4.0	5	5.5	5.0	5
2012/2013	4.9	4.5	5	3.6	3.0	4	5.9	5.5	6

Lecturers of the Department of Mathematics developed activities in e-environment. Apart from theoretical materials students get the problems for individual work, also they can see examples for doing individual work step-by-step and how to solve these problems in the mathematical software “MathCad”. Students have an opportunity to use e-environment to get more examples which the system generates. In this way students can have similar problems for solving. The individual work is compulsory for students, but they can do more generated problems for self-learning purposes before tests during the semester or before the final exam.

Interactive self-assessment theory tests were created in e-environment in the last study year. It was quite a time-consuming process. Comparing the study results in mathematics during the last four years of the students of the Faculty of Rural Engineering, dynamics is positive, as it can be seen from Table 2.

Table 2

Study results of theory tests, tasks tests and the exam in Mathematics of students from the Faculty of Rural Engineering

Study years	**Theory test**			**Tasks test**			**Exam**		
	Mean	*Me*	*Mo*	*Mean*	*Me*	*Mo*	*Mean*	*Me*	*Mo*
2009/2010	3.5	3.0	3	3.2	3.0	3	4.3	4.0	4
2010/2011	4.6	4.0	4	4.4	4.0	4	4.5	4.0	4
2011/2012	4.7	5.0	5	4.6	4.5	4	4.8	4.5	5
2012/2013	5.2	5.0	5	5.0	4.0	5	5.2	5.0	5

The results of the theory test in 2011/2012 study year, when the theory tests were written three times, improved by about 16 %, the results of task tests improved by about 21 %, but the results of the final exams were better by about 9 % comparing with 2009/2010 and 2010/2011 study years. In 2012/2013 study year the theory tests were written three times, but students in addition could use self-assessment theory tests in e-environment; the results of the theory test improved by about 32 %, the results of task tests improved by about 31 %, but the results of the final exams were better by about 21 %, comparing with the previous two years.

The latest results of our research show that students prefer to use this method of knowledge consolidation.

Conclusions

The theoretical and practical research shows that the theoretical knowledge in Mathematics is important for development of competences necessary for engineers: firstly, the skill to apply formal rules exactly, secondly, the skills to use language of symbols, thirdly, the skills dealing with reduced attrition (mental and physical) reducing cognitive load using the method of substitution, as well theoretical knowledge is necessary for solving different kinds of engineers’ problems.

The study of theory in the study course of Mathematics takes only 25 % of all the study time. Students seldom learn theory enough before the exam. One theory test at the end of the semester got low grading. The situation grew better when the theory test was dividing into three parts. Students prepared for tests more and better in this case. Using e-environment also helped to get better results. The self-assessment theory tests had positive influence on the final exam assessment. The results of the theory test improved by approximately 25 %, the assessment results of the task tests – by about 30 %, the final exam assessment results improved by about 21 %.

The requirement was included in the study process stating that task tests are allowed to be taken only if students have done individual works (step-by-step and MathCad) for the respective theme. At the same time students cannot use the interactive self-assessment theory test before the theory test. Further to improve the results of the final exam the department staff should implement additional requirements: students should do compulsory self-assessment tests in e-environment before theory and task tests.

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